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**Artifact Reduction with Diffusion Preprocessing for Image Compression**

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**Abstract**

We evaluate a preprocessing method for image compression artifact reduction based on non-linear diffusion filtering that we proposed earlier. The method consists of using edge-adaptive diffusion processes before the DCT-JPEG compression. By using a simple measure for artifact reduction, we show that a considerable artifact reduction is achieved with preprocessing at the same bit rate as and with no greater error than the original compression. We also show that preprocessing helps to preserve the true contours for image processing applications. An automatic parameter selection for the preprocessing is also proposed considering the edge-histogram of the image, and depending on the compression ratio. We tested the method for visual quality with extensive subjective measurements. We show that depending on the image content preprocessing can significantly improve the visual quality at low bit rates.

**Keywords:** image compression, compression artifact, scale-space, anisotropic diffusion, subjective quality, preprocessing, contour-validity.

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## 1 Introduction

The quantization error in image compression appears in the form of typical patterns such as ringing around the edges, false texture, or visible block-boundaries in block-partitioning schemes [32]. Such patterns are called compression artifacts. In a series of previous papers [16,17,34] we proposed the use of non-linear diffusion preprocessing for reducing the artifacts.

Images with numerous details can degrade more than those with fewer details when being compressed at the same bit rate [32]. Preprocessing the image before compression can alleviate the components of the image susceptible to artifacts. For example, transform based compression schemes suffer from ringing artifacts due to the sharp frequency cutoff involved in the raw quantization. Filtering the image before compression alleviates the sharpness of the cutoff and reduces this artifact.

Preprocessing has the advantage over other artifact reduction methods like postprocessing [32] and visual weighting [35,39] that it does not require a change in the decompression, and compression standards remain unaffected. Moreover, preprocessing is done only once, while any postprocessing must be done each time the image is decompressed, involving additional computational complexity. Some prefiltering options already exist in image compression software products (e.g. in [41]) and are used in sharpness/noise optimization of digital cameras.

In [34] and [16] we proposed the use of non-linear diffusion as a preprocessing step before compression to reduce the blocking artifacts in DCT-JPEG. Non-linear diffusion was introduced in [28] as an adaptive image filtering method attempting to preserve the main edge-structure in the image. Due to this property, the artifacts can be reduced while preserving the main structures important for visual perception or image processing algorithms. The preprocessing method in its final form was given in [17], where it is described as local adaptive modification of Gaussian filtering. Some kind of adaptive filtering that alleviates the artifacts and enhance the edges in order to improve the visual quality of the compressed image is frequently used by postprocessing methods [4,6,20,31,32].

In this paper, we extend the results given in [17] by proposing new parameter selection strategies and more comprehensive visual tests in order to draw final conclusions about the method.

Our results relate to DCT-JPEG, due to our previous research and due to the simplicity of interpretation in this case, but the same methodology could also be considered for compression schemes with similar artifacts.

We organized the paper as follows. After briefly considering the previous work done on or related to preprocessing, we define the adaptive filters used in preprocessing by generalizing the Gaussian filtering and give a simple way of measuring and expressing artifact reduction through the peak signal to noise ratio (PSNR) as introduced in [17]. This measure is used for localizing the useful parameters for diffusion preprocessing. We compare the efficiency of the preprocessing with different parameters by analyzing the edge detection results for the compressed images, and by using visual tests for evaluation, since at low bit rates our images have a high level of degradation and we cannot merely rely on the PSNR results.

We obtained that though the maximal improvement by diffusion preprocessing in terms of PSNR is only about 0.1 to 0.4 dB, the artifact patterns are significantly reduced for the filtered images. The preprocessed images can give better edge detection results and depending on the image even a significantly better visual quality.

## 2 Related Work

Compressions artifacts result from the interaction between the quantization and the image content. Though it has been recognized [32] that preprocessing can improve the quality by reducing these effects, this problem has not been considered in a systematic way. Some results are available for video coding, where it is a well established practice to filter before quantization [22,15]. The most disturbing artifacts in compressed video are due to motion compensation errors, thus filtering the motion compensated pictures before quantization improves the quality [22]. These are not “real” preprocessing methods, since they have to be repeated during decoding, which can increase the decoding time – e.g. up to 30 percent for H.264 decoder [22]. The repetition can be avoided by applying the preprocessing to the displaced picture differences instead of the motion compensated pictures. This approach was proposed in [15] for MPEG-2 video. In both approaches, the strength of

the separable [22] respectively isotropic Gaussian [15] filter is locally adjusted to reduce the blocking and to preserve the real edges. Though these methods process the “intra” (non-motion compensated) pictures as well, which is actually related to the preprocessing of still images, the largest improvements in video preprocessing are due to the processing of the motion predicted frames [22]. Therefore, the results for video coding do not directly apply to still image compression. Additionally, artifacts like blocking in video can be visually much more disturbing than in still images due to temporal visual effects. It should be noted that one of the results of this paper is the advantage of non-linear/non-isotropic smoothing against the linear/isotropic one above.

Preprocessing is strongly related to the problem of lossy compression of noisy images [1]. In this framework, the images are degraded with well-defined noise models such as Gaussian, Poisson, or film grain noise. Results from information theory grant that a rate-distortion optimal solution is obtained by cascading the optimal estimation of the original image with a conventional coder [4] like JPEG [1]. The optimal estimation in [1] is achieved by a Markov-Random-Field (MRF) regularization to grant edge-adaptivity. The problem with this approach is that MRF models have large complexity and it is difficult to establish the model especially if the noise is unknown. The same problem was considered in [9] by using total variational (TV) image regularization instead of MRF before wavelet compression. Preliminary results with an artificial test image are shown there. Similar results with an artificial image and added noise are also found in our paper [34]. Note that there is an essential difference between evaluating the results of compression with noise and the results of artifact reduction. For the former, the compression results can be compared to the original noise-free images [1, 34]. When evaluating artifact reduction on standard test images, however, there are no “originals” given as references. In this paper we circumvent this problem by visual testing, where the compression results are directly compared in the absence of original reference images.

The most important question in preprocessing is how to choose the strength of the preprocessing filter. In [1] this is determined by the noise model. For video coding, the strength of the filter usually depends on the type of the frame (“intra” or predicted) and the quantization parameter [22]. A criterion based on distortion-rate optimization is proposed in [15]. The variance of the Gaussian filter is chosen for each macro-block so that the Lagrangian cost function including the distortion and the encoding

costs is minimized. In our approach, the diffusion model as described in the next section grants the local adaptability of the filtering. The strength of the preprocessing is selected based on distortion rate considerations. However, we have found that in contrast to the results in video processing, the maximal improvement in PSNR is likely to be insufficient to achieve a large artifact reduction or a significant visual quality improvement. For this reason, we apply alternative parameter selection rules and use visual tests for the evaluation.

### 3 Non-linear Diffusion and Adaptive Filtering

The theories considering the application of partial differential equations in image processing [2, 11, 21, 28] explore adaptive filters acting locally and accounting for specific structures (edges, level-lines, etc.) in the image. These filters should keep the information (e.g., edges) meaningfully at the specified scale, and filter out the disturbing details at lower scales. It has been shown [2,12,21] that filtering operations verifying certain simple invariance, regularity, and locality assumptions are indeed guided by linear or non-linear diffusion processes.

We consider diffusions that are potentially useful for artifact reduction having different levels of edge-adaptability. We begin by explaining the *linear diffusion* (LD) process. Some of the following notations are found in the Appendix. We treat images as positively valued smooth functions defined at the points  $\mathbf{x} \in \Re^2$ . Let us take the smoothing of an image  $f$  with Gaussian kernels

$$G_t(\mathbf{x}) = \frac{1}{4\pi t} \exp\left(-\frac{|\mathbf{x}|^2}{4t}\right), \quad \mathbf{x} \in \Re^2, \text{ having various parameters } t > 0. \text{ We obtain a family of images}$$

$(u_t)_{t \geq 0}$  with  $u_0 = f$  and  $u_t = G_t * f$ , where “\*” denotes the convolution. The family consists of images, which lose details but retain information on larger scale features with the increasing parameter  $t$ . An element  $u_t$  of this family can also be obtained [11,21] by a linear diffusion process done on the image up to time  $t$ . The latter means that  $u_t(\mathbf{x}) = u(t, \mathbf{x})$  for all  $t > 0$  and  $\mathbf{x} \in \Re^2$ , where  $u$  is the solution of the LD equation

$$\partial_t u = \Delta u, \tag{1}$$

**Table 1** Diffusions with different degree of adaptability.

	Directionally insensitive (isotropic) $\alpha = 0.5$	Directionally adaptive (anisotropic) $\alpha = 0$
Contrast insensitive $K = \infty$	Linear Diffusion (LD)	Mean Curvature Motion Diffusion (MCMD) [2]
Contrast adaptive $0 < K < \infty$	Non-linear Isotropic Diffusion (NLID) [30]	Pure Anisotropic Diffusion (PAD) [3,8]

where  $\Delta$  is the Laplacian operator, and the initial condition is  $u_0 = f$ . This is why the LD is said to generate multiscale representations of images [2,11] and the main diffusion parameter  $t$  is called the scale or the scale parameter of the diffusion.

To understand the underlying filtering  $\Delta u$  is written as a sum of two orthogonal components

$$\Delta u = u_{\parallel} + u_{\perp}, \quad (2)$$

where  $u_{\parallel}$  denotes the second spatial derivative in the direction orthogonal to the gradient  $\nabla u$ , and  $u_{\perp}$  is the component in the direction parallel with the gradient  $\nabla u$ . By noting that  $u_{\parallel}$  and  $u_{\perp}$  are one dimensional analogs of  $\Delta u$ , and under the assumption that the gradient gives a rough estimate for the strength and the direction of edges,  $u_{\parallel}$  and  $u_{\perp}$  can be interpreted as “infinitesimal” Gaussian filterings along and across the edge. This low-pass filtering can contribute to the reduction of the ringing artifact, which is the result of the sharp frequency cut-off caused by quantization at lower bit rates. In Eq. 2, the two directional terms have equal weights, and both depend only on the local direction  $\nabla u^{\perp} / |\nabla u|$  of the edge and not on its local contrast  $|\nabla u|$  (see Appendix).

To add contrast and directional sensitivity, Eq. 1 can be extended to the general form

$$\partial_t u = p(|\nabla G_{\sigma} * u|)u_{\parallel} + n(|\nabla G_{\sigma} * u|)u_{\perp}. \quad (3)$$

where  $p(\cdot)$  and  $n(\cdot)$  are weighting functions controlling the diffusion along and across the edges, respectively, and  $\sigma > 0$  is a fixed parameter. The values of  $p$  and  $n$  depend on the norm of the gradient of the smoothed image. The purpose of the pre-smoothing [28] with  $G_{\sigma}$  is to obtain a reliable estimate on edges, and to make the equation robust against noise. The idea is to allow full diffusion at uniform

regions where the value of  $|\nabla G_\sigma * u|$  is small and to inhibit the diffusion at edge locations where  $|\nabla G_\sigma * u|$  is large. One possibility to control the diffusion in this way is to use the weighting function

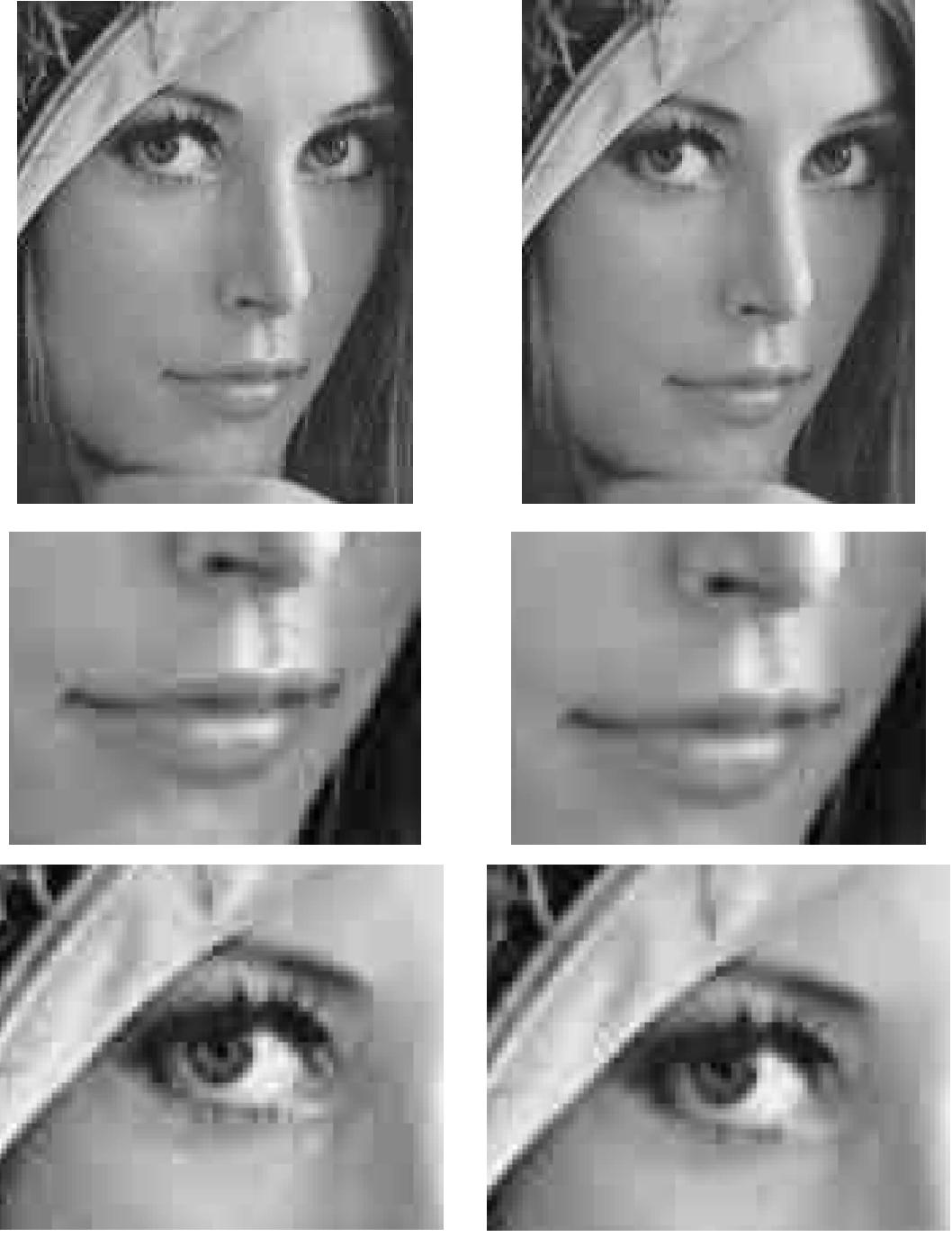
$$w_K(x) = \begin{cases} 2 \exp\left(-\frac{|x|^2}{K}\right) & , \quad 0 < K < \infty, \\ 2 & , \quad K = \infty \end{cases} \quad (4)$$

where  $x \in \mathbb{R}$  and  $K \in (0, +\infty]$  is a fixed parameter [28]. With the special choice  $p = (1 - \alpha)w$  and,  $n = \alpha w$  where  $0 \leq \alpha \leq 0.5$ , we obtain the diffusions examined in this paper

$$\partial_t u = w_K(|\nabla G_\sigma * u|)((1 - \alpha)u_{\parallel} + \alpha u_{\perp}). \quad (5)$$

The diffusions obtained for particular choices of the parameters are listed in Table 1. Proofs of existence and uniqueness of the solutions and stable numerical schemes are given for these choices of parameters [3]. We note that the MCMD is a morphological operation, since it is invariant for an arbitrary monotone contrast modification [2].

Whatever the parameters  $\alpha$  and  $K$  are, the diffusion can contribute to the suppression of the ringing artifact to some degree (if  $\alpha > 0$ ), as explained above for the LD. Moreover, since the minima and the maxima of the intensity values get closer, the DC values of the neighboring DCT blocks of a flat area are more likely to fall into the same quantization bin after the preprocessing, thus decreasing the blocking artifact. Note that in this way artifact reduction is achieved also for images without a lot of high frequency content. An example is given in Fig. 1 for the test image ‘‘Lena’’.



**Figure 1** Detail of Lena (512×512) at 0.4 bpp, and two magnified small fragments. Blocking is reduced in the low frequency regions (face), ringing is reduced around edges (mouth, nose, eyes).

We used forward numerical schemes for the above equations in all of our experiments with the fixed step size  $\lambda = 0.1$  and  $\sigma = 0.4$ , by recasting  $(1-\alpha)u_{\parallel} + \alpha u_{\perp} = \alpha \Delta u + (1-2\alpha)u_{\parallel}$  and computing  $u_{\parallel}$  as in [3]. The smoothing with  $G_{\sigma}$  for the gradient computation was done with LD. The scale values

indicated were computed as  $t = \lambda m$ , where  $m$  is the number of iterations done by the numerical scheme.

## 4 Artifact Reduction

To explain artifact reduction, let us observe the details of the different compressed versions of the test image “Goldhill” shown in Fig. 2 first. We can see that artifacts become less visible by preprocessing, if we compare the images along high-contrast edges and on surfaces like walls and roofs. Motivated by this, we try to define and measure artifact reduction.

Let  $P_t$  denote a diffusion preprocessing method done up to scale  $t$  and  $C$  the compression method (JPEG in our case). We assume for now that the bit rate is fixed. We say that an image  $f$  can be compressed at better quality than an image  $g$  at the given rate if  $PSNR(C(f), f) > PSNR(C(g), g)$ , where  $C(f)$  and  $C(g)$  denote the compressed images. Our goal is to transform  $f$  into an image  $f'$  by preprocessing so that we can compress  $f'$  at a better quality than  $f$  under the constraint that the compressed image  $C(f')$  remains acceptably close to  $f$ .

The reconstruction quality of the preprocessed image  $P_t f$  at the given fixed bit rate will be

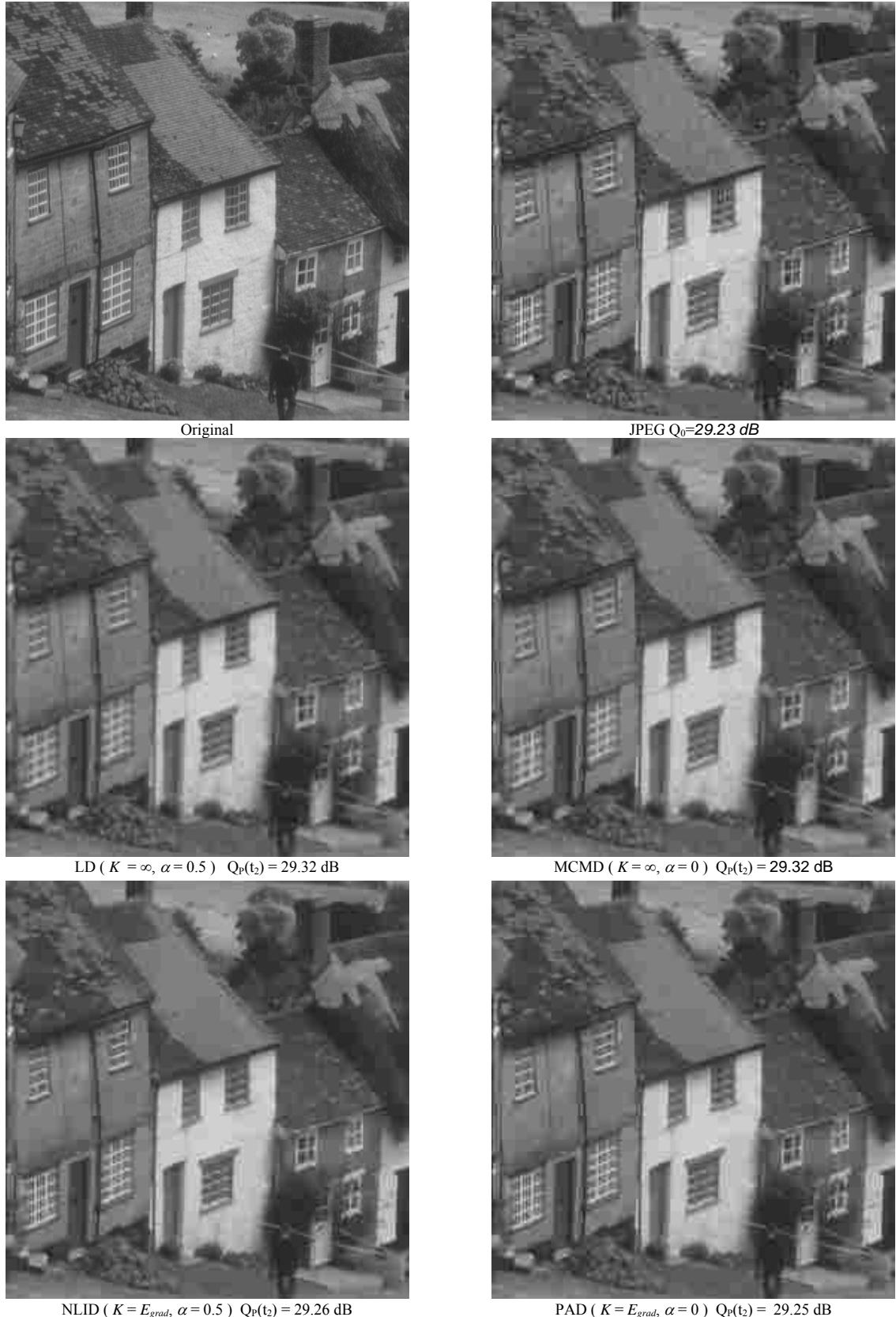
$$Q_{PP}(t) = PSNR(C(P_t f), P_t f).$$

The quality of the compression with preprocessing relative to the original image  $f$  is

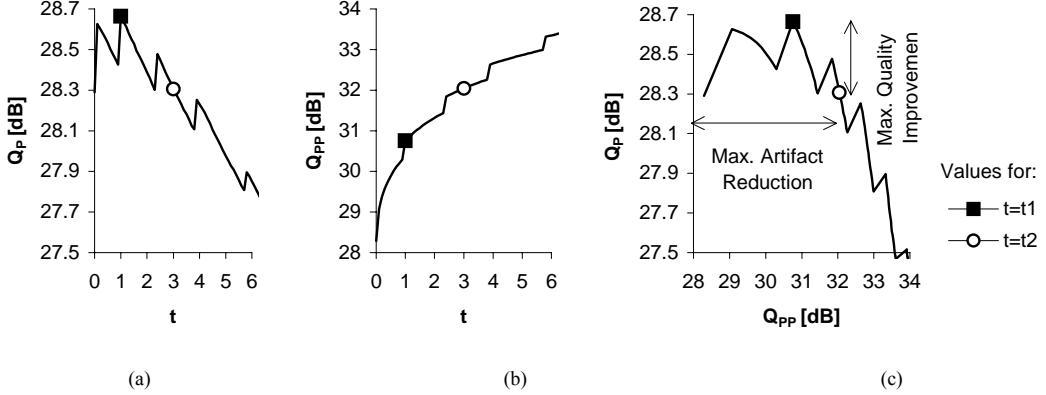
$$Q_P(t) = PSNR(C(P_t f), f).$$

The quality of the compression without preprocessing is  $Q_0 = Q_{PP}(0) = Q_P(0) = PSNR(C(f), f)$ .

A diffusion processing method is said to reduce artifacts if there is a scale  $t \geq 0$  such that  $P_t f$  can be compressed at a better quality than  $f$ , i.e., if  $Q_{PP}(t) > Q_0$ , under the constraint  $Q_P(t) \geq Q_0$ . The latter constraint means that any processing is meaningful only if the original quality does not decrease.



**Figure 2** Preprocessing for “Goldhill” ( $c = 0.25$  bits/pixel) using maximal artifact reduction ( $t = t_2$ ).  $E_{grad} = E(|\nabla G_\sigma * f|)$  is the average gradient magnitude.  $Q_P$  and  $Q_o$  are the qualities compared to the original.



**Figure 3** Preprocessing for “Goldhill” with NLID ( $c = 0.2$  bits/pixel,  $K = 0.0237$ ). (a) the PSNR values versus the original image  $Q_P(t) = \text{PSNR}(C(P_t f), f)$ , (b) the PSNR values versus the processed image  $Q_{PP}(t) = \text{PSNR}(C(P_t f), P_t f)$ , (c) the curve  $t \mapsto (Q_{PP}(t), Q_P(t))$ .

The tendencies of the curves  $Q_P$  and  $Q_{PP}$  as a function of  $t$  are shown for “Goldhill” in Fig. 3a and 3b for the preprocessing with NLID (Table 1) at bit rate  $c = 0.2$  bits/pixel. The combined plot of  $Q_P(t)$  and  $Q_{PP}(t)$  as a function of the increasing scale  $t \geq 0$  is shown in Fig. 3c. The plots were obtained by preprocessing the images up to a given scale, and then by compressing them to the target bit rate with JPEG [41]. The target bit rate was reached with a trial-and-error procedure by successively changing the quality parameter of the compressor. The fixed bit rate  $c$  was achieved with an error of 1.5% in average for all diffusion methods and images. Note that  $Q_{PP}(t)$  increases monotonously with  $t$ , and that we can use it to measure artifact reduction (under the constraint  $Q_P(t) \geq Q_0$ ). The qualitative properties of these curves were the same for all diffusion methods and test images we tried.

We define the following two characteristic scales for each particular preprocessing method  $P$ :

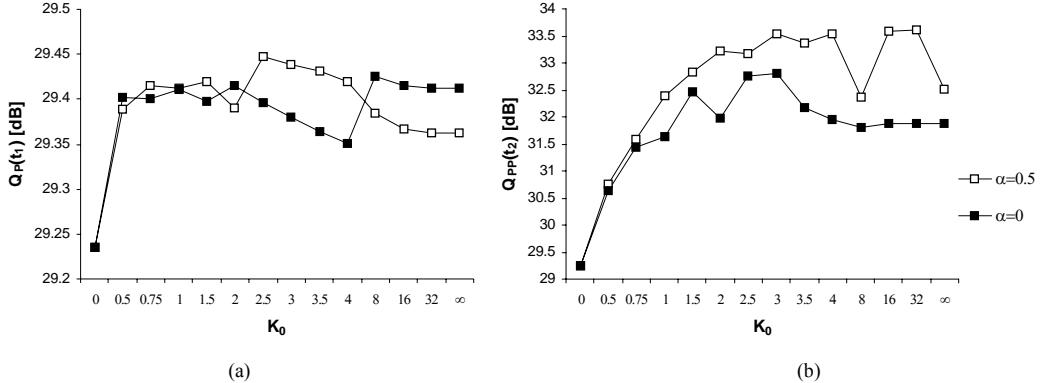
1. The scale corresponding to the maximal quality improvement,

$$t_1 = \max \arg \max \{Q_P(t) \mid t \geq 0\}. \quad (6)$$

2. The scale corresponding to the maximal artifact reduction,

$$t_2 = \max \{t \geq 0 \mid Q_P(t) \geq Q_0\}. \quad (7)$$

These scale values are indicated in Fig. 3. Note that preprocessing up to scale  $t_2$  will redistribute the original compression error, so that the maximum portion of the quality will be devoted to the main



**Figure 4** The values of  $Q_P(t_1)$  and  $Q_{PP}(t_2)$  for  $P = P^{\alpha,K}$ , where  $K = K_0 E(|\nabla G_\sigma * f|)$ .  $K_0 = 0$  means no preprocessing. The image was “Goldhill” ( $c = 0.25$  bits/pixel).

structure of the image, as defined by the underlying multiscale representation, and the smallest portion of quality will be allotted to the small-scale details and noise. Different diffusion methods will do this redistribution in different ways. According to the theory concerning the filtering with non-linear diffusion, the more adaptive the diffusion is, the better this redistribution will be. The results of the contour validity in Section 5.2 and subjective tests in Section 6 support this claim.

Compression results for preprocessing at  $t_2$  for “Goldhill” with different diffusion methods are shown in Fig. 2. Visibility of the block boundaries is reduced and false patterns within the blocks are eliminated. The corresponding PSNR values are found in Table 4.

The scales  $t_1$  and  $t_2$  are image dependent. Since any diffusion involves slight changes in the dynamic range of the image, it is sometimes useful to use the same processing scale for a group of images, e.g., in video coding. This will also reduce computing time. To do this, parametric models can be used to select the appropriate processing scale [16]. We shall discuss this later on.

## 5 Testing the Diffusions by Objective Measures

We have tested the importance of contrast and directional control for preprocessing with respect to the PSNR quality and artifact reduction measures defined in the previous section.

Let  $P^{\alpha,K}$  denote the diffusion process guided by Eq. 5 with  $K \in (0,+\infty]$  and  $\alpha \in \{0,0.5\}$ . We computed the quality value  $Q_P(t_1)$  and the artifact reduction  $Q_{PP}(t_2)$  with  $P = P^{\alpha,K}$ , where we altered the values of  $K$  and  $\alpha$ . The results for the test image “Goldhill” at 0.25 bits/pixel are shown in

**Table 2** The quality improvement and artifact reduction achievable at scale  $t_1$ , and the artifact reduction achievable at scale  $t_2$  (statistics for eight typical  $512 \times 512 \times 8$  bit grayscale photographic images: “Goldhill”, “Boat”, “Bridge”, “Lena”, “Barbara”, “Man”, “Frog”, “Elaine”, “Pepper”).

c [bits/pixel]	$\max_p Q_p(t_1) - Q_0$ [dB]			$\max_p Q_{PP}(t_1) - Q_0$ [dB]			$\max_p Q_{PP}(t_2) - Q_0$ [dB]		
	Average	Standard Deviation	Max	Average	Standard Deviation	Max	Average	Standard Deviation	Max
0.2	0.22	0.17	0.43	2.73	1.49	4.44	3.76	2.14	5.92
0.25	0.22	0.10	0.38	2.83	1.17	4.33	3.97	1.89	6.28
0.3	0.21	0.08	0.33	3.06	0.90	4.39	3.79	1.23	5.24
0.35	0.16	0.09	0.32	2.83	0.93	4.74	3.52	1.40	5.83
0.4	0.17	0.09	0.35	2.92	0.80	4.39	3.73	1.64	6.37
0.45	0.13	0.05	0.19	2.67	1.24	4.88	3.11	1.54	5.99
0.5	0.11	0.06	0.22	2.56	0.84	3.96	2.76	1.15	5.06

Fig. 4. The parameter  $K$  was always a multiple of the average gradient magnitude  $E_{grad} = E(|\nabla G_\sigma * f|)$ , computed for the original image  $f$ .

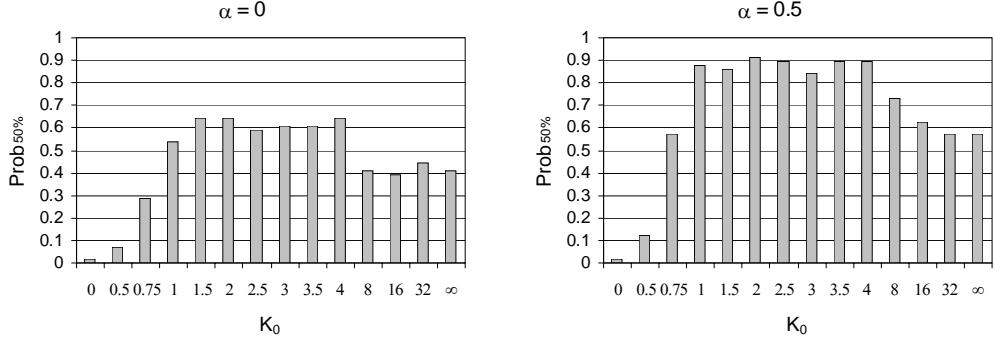
We can see from Fig. 4a, that the maximal PSNR quality improvement  $Q_P(t_1)$  obtainable by diffusion preprocessing is quite small. Though results for other images and bit rates have shown that it is possible to improve the PSNR with 0.1 to 0.4 dB, depending on the image and bit rate. For some of these images 0.5 to 0.9 dB improvement was reported for similar bit rates with postprocessing methods [32].

Maximal artifact reduction is shown in Fig. 4b and it is considerable. We obtained that the maximal values of  $Q_{PP}(t_1) - Q_0$  are between 2 and 4 dB, and the maximums of  $Q_{PP}(t_2) - Q_0$  are between 2 and 6 dB, depending on the image and bit rate. This shows that preprocessed images can be compressed with up to 4 dB better quality than the original images at the preprocessing scale  $t_1$ , and with up to 6 dB better quality at the preprocessing scale  $t_2$ , without loss in quality as compared to the original image. These results are summarized in Table 2.

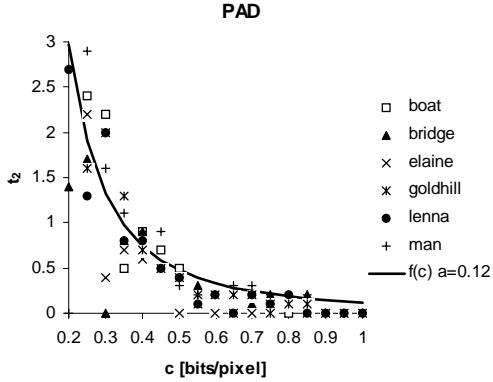
### 5.1 Choosing the preprocessing parameters

Though there are some tendencies in the curves of Fig. 4b, we do not get a method  $P^{\alpha,K}$  that would clearly qualify as the best one. How can we find the “good” parameters  $K$  and  $\alpha$  achieving a relatively large artifact reduction with a high probability?

A statistical analysis of the artifact reduction data yields a clearer picture. For each image and bit rate, we computed  $Q_{max} = \max_{\alpha,K} Q_{P^{\alpha,K} P^{\alpha,K}}(t_2)$ , the largest possible maximal artifact reduction using



**Figure 5** Probabilities of obtaining large values  $Q_{PP}$  for  $P = P^{\alpha,K}$ ,  $K = K_0 E_{grad}$ ,  $E_{grad} = E(|\nabla G_\sigma * f|)$ .



**Figure 6** Using parametric models for  $t_2$ .  $K = E_{grad}$ ,  $f(c) = ac^{-2}$ .

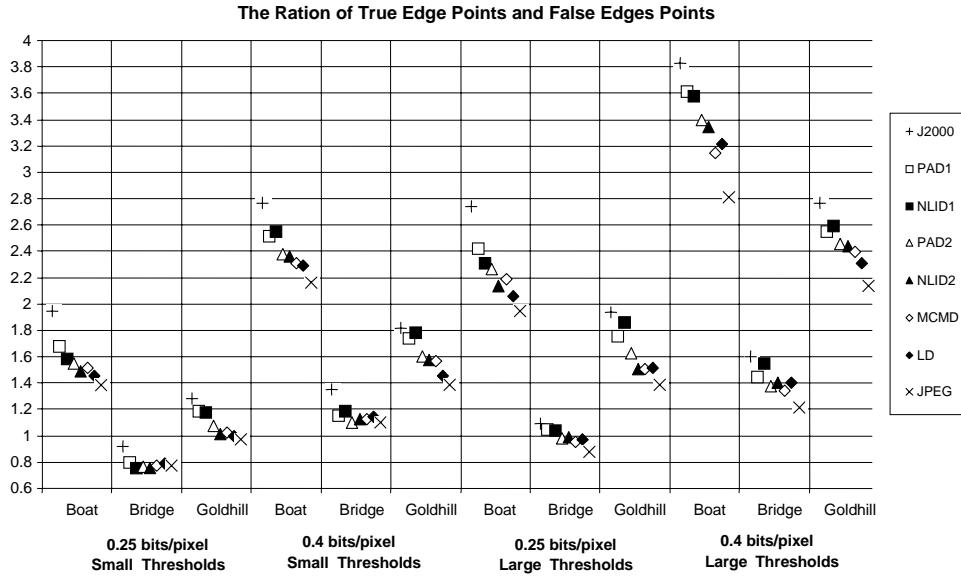
diffusion preprocessing. By taking a fixed method  $P = P^{\alpha,K}$  we are interested in the probability  $\text{Prob}_{50\%}(\alpha, K)$  that  $Q_{PP}(t_2) \geq (Q_{\max} + Q_0)/2$ , i.e., that  $P$  will result in an artifact reduction yielding at least the half of the maximum possible value relative to  $Q_0$ . We obtain a raw estimate of these probabilities by taking a number of images and bit rates, and counting how many times is  $Q_{PP}(t_2)$  greater than  $(Q_{\max} + Q_0)/2$ .

The probabilities obtained this way are shown in Fig. 5 for different values of  $\alpha$  and  $K$ . The values in Fig. 5 suggest that the largest maximal artifact reduction values tend to be obtained by isotropic ( $\alpha = 0.5$ ) diffusions with  $E_{grad} \leq K \leq 4E_{grad}$ . However, we shall show later that for contour validity, which is not measured by  $Q_{PP}$ , the case  $\alpha = 0$  becomes important.

We briefly discuss the automatic choice of the parameter  $t_2$  now. As mentioned previously, parametric models can be used to compute the number of iterations for the maximal artifact reduction. We discuss an example for PAD. In Fig. 6 we show the values  $t_2$  for a set of images, obtained with a

**Table 3** The diffusion processes used in the edge detection example and in the visual tests.  $E_{grad}$  is the average of the gradient magnitudes of the original image.

K		$\alpha$	
		0	0.5
		$\infty$	MCMD
	$3E_{grad}$	PAD <sub>2</sub>	NLID <sub>2</sub>
	$E_{grad}$	PAD <sub>1</sub>	NLID <sub>1</sub>

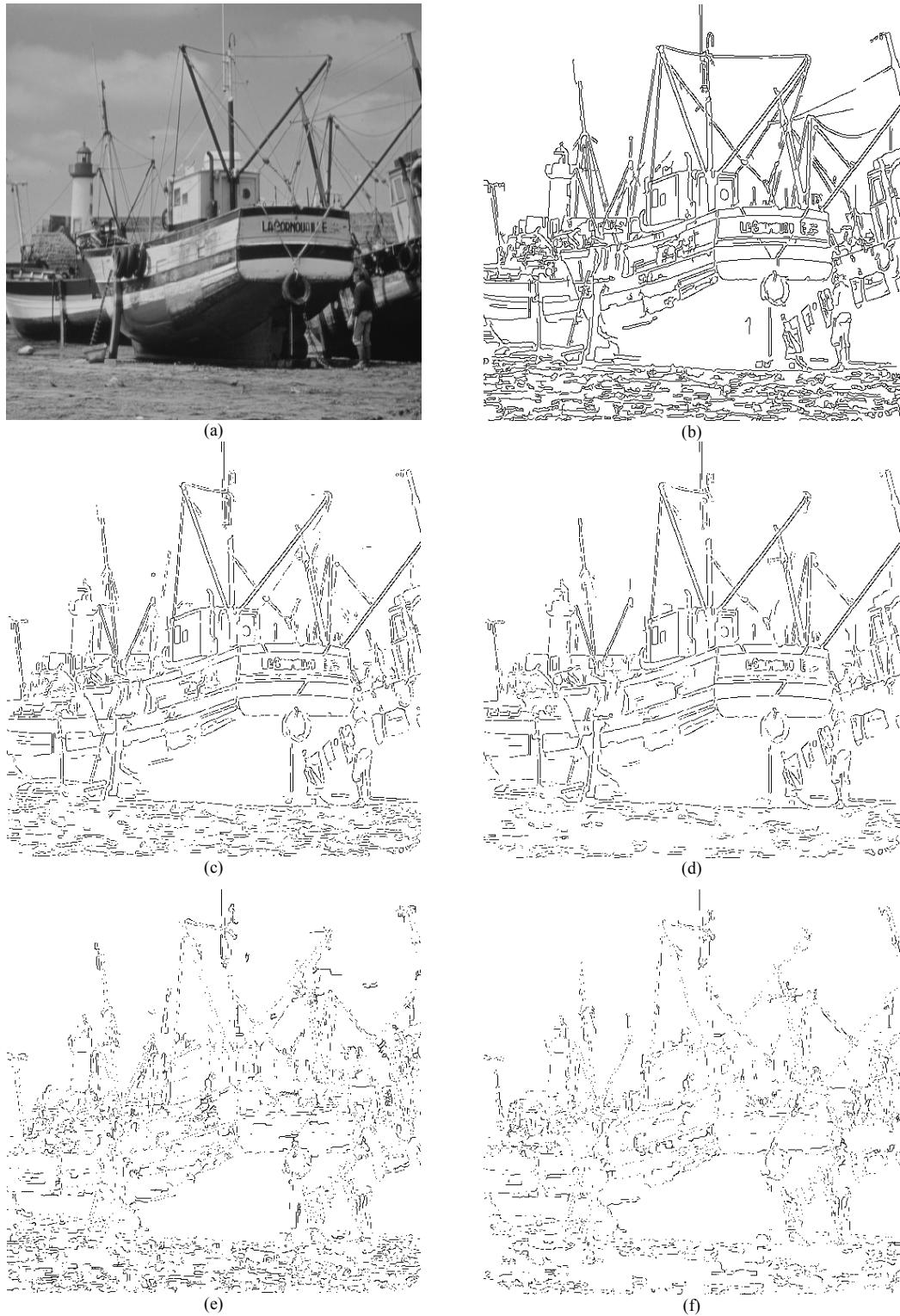


**Figure 7** Results for the edge detection. The ratio of the number of true edge points and the number of points that are detected but are not part of the contours in the original image.

search according to Fig. 3 (there may be some noise in the data, due to the fluctuation of the curve in Fig. 3c). We show a fitting function  $f(c) = ac^{-2}$  for ‘Goldhill’. This can be used as an average curve approximating  $t_2$  for other images as well. The value ‘ $a$ ’ like the quality parameter of JPEG is a compression parameter, which should be adjusted to the respective purpose.

## 5.2 Quantifying the edge preserving property

To illustrate and quantify the edge preserving property of non-linear diffusions, and to show how artifacts impair the stability of image processing algorithms, we did a series of tests for edge detection. We sampled the space of the preprocessing parameters and tested six different combinations. These are listed in Table 3. The edge maps were extracted for the JPEG compressed images with and without preprocessing, and the so obtained contours were compared to those extracted from the original image. The preprocessing was done up to the maximal artifact reduction.



**Figure 8** Edge detection for Boat, 0.25 bits/pixel, using small thresholds. (a) The original image, (b) Contours of the original, (c) True contour points for JPEG, (d) true contour points for JPEG with  $PAD_1$  preprocessing, (e) false detections for JPEG, (f) false detections for JPEG with  $PAD_1$  preprocessing.

Edges were detected with an algorithm [24] based on Canny's definitions for the edges [7]. An edge point is assumed to be located where the response of a filtering with a derivative of Gaussian (DoG) filter in the direction of the gradient obtains its maximum. The final edge map is obtained by using hysteresis thresholding for these maxima and connectedness search.

The same parameters were used for all images (the DoG parameter was 1, the minimal size of the connected components was 20). There were two different sets of parameters; one with small threshold values (Low = 10, High = 20), and one with large threshold values (Low = 20, High = 30).

The reliability of edge extraction results was measured by computing the ratio of the number of true edge points detected and the number of false detections, i.e., those points, which were detected as edges, but are not edges on the original image. This ratio is shown in Fig. 7 for JPEG, for JPEG with different preprocessing methods and for JPEG2000. Clearly, the more contrast adaptive the diffusion is, the better the reliability of the edge extraction will be.

The true and false edge points are shown in Fig. 8 for the image "Boat". Though the compression without preprocessing results in more true edges detected, the proportion of false edge points is also larger, and there are more connected structures appearing as false edges. Note that there is no way of deciding whether an edge point is true in the lack of the original image. A poor ratio of true and false edges can therefore make the edge map unreliable. Results show that the main contours are not changed after preprocessing, but the noisy textured area has been partly removed.

## 6 Subjective Testing

Save for a few exceptional cases, e.g., OCR [33] or video compression with a hypothetical additive degradation model [38] or error patterns [25], it is in general difficult to find all physical properties of images that correlate with perceptual quality. We know of attempts to find and standardize perceptual quality metrics and measurement procedures [10,19] to improve the design of compression algorithms. Though it has been shown [18] that for higher bit rates, PSNR correlates well with the human assessments, at low bit rates, compressed images suffer from severe artifacts and PSNR does not correlate well with them, which will be shown through our results as well. Even more complex perceptual metrics incorporating visual response models [19] are mostly appropriate only for ordinary

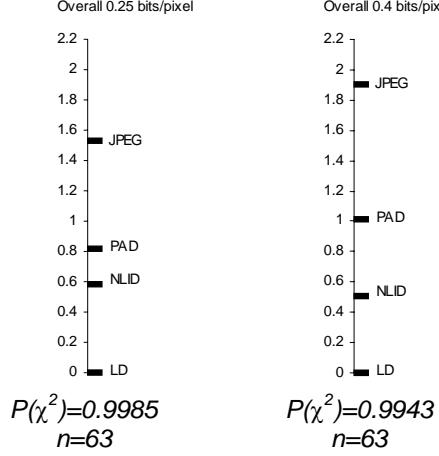
bit rates. These methods, parameterized by the viewing angle and resolution, are based on Gabor-channel filtering and weighting [23]. In a previous work [16], we have tested a similar non-standard method of [19] for the parameter estimation of preprocessing. Thus, to obtain the reliable rating of compression methods concerning visual quality, we use subjective tests.

Instead of asking the test person for direct rating [42] of images on an absolute quality scale (e.g., by giving scores between 1 to 5), it is much easier to make the test person compare images on similar level of degradation, and ask her or him to choose the better one. A scale can be constructed based on these choices. Pair comparisons were used in a construction of the so-called JND scale for video impairment [38], also proposed for an IEEE draft standard [10].

Although we have shown that it is possible to obtain a maximal improvement in PSNR compression quality by preprocessing up to scale  $t_1$ , it is, however, quite small and will not yield a large perceivable artifact reduction in general. With the maximal artifact reduction  $t_2$ , there is a larger perceivable artifact reduction. PSNR compression quality of the images will be a little bit greater or equal to that of the original JPEG compression in this case. We were therefore primarily interested in comparing the different preprocessing methods on a subjective scale for the maximal artifact reduction (scale  $t_2$ ). We did a so-called Thurston-scaling experiment, referenced as *the Case V. of Thurston's law of comparative judgment* [36,37].

### 6.1 The construction of the subjective scale

The goal is to order  $m$  items (the different compression methods) on a one-dimensional scale according to a criterion (e.g., subjective assessment of “goodness”). The test persons do choices between all possible pairs of items. The relative frequency of a choice for a given pair gives the probability of preferring one item to the other one. The scale is constructed based on these probabilities, which gives a one-dimensional representation for these pair preference relations. The Thurston scale construction assumes that if an item is fixed as a zero position on the scale, a test person is able to place the remaining items relative to the fixed one with a given uncertainty. The placement fits to a Gaussian distribution where the position of an item is the expected value of the placement, and the uncertainty is its variance. The uncertainty is assumed to be equal for all items, so it can be chosen as a unit for the scale. There are situations where the obtained scale is not reliable. For



**Figure 9** Scaling results for NLID, PAD, and LD.  $K=0.05$  ( $2E_{\text{grad}} \leq K \leq 3E_{\text{grad}}$ ). Overall scales obtained by collecting the scores for the different methods over the three test images [17].

example, if the pair relations cannot be represented in one dimension, or if an outlier is present in the measurements (which gives an almost certain choice). The reliability of the scales can be tested with the  $\chi^2$ -test of Mosteller [26], typically used for these purposes.

The pair comparison of two images can be done in two different ways. They can be compared directly or in the presence of the original image. If we construct a subjective scale measuring the distance compared to the original image, we do the comparisons in the presence of the original one. This would give the subjective analog of  $Q_P$ . We refer to this as “*the scaling experiment with three images displayed*” (SE3). By excluding the original image from the display, the test person will choose the image that she or he judges as looking better, based on an impression of quality without taking reference to the original image. We refer to this as “*the scaling experiment with two images displayed*” (SE2).

During the visual experiments we tested the three images analyzed in Figure 7. Two of them (“Boat” and “Goldhill”) are balanced “normal” images with sharp edges, smooth sections and textured areas. The third image (“Bridge”) is an overly textured one. Except for the sharp contour of the planking, the whole image is filled by different textures. Contour detection (ratio of true and false edges in Figure 7) has not been improved by preprocessing for this one, differently from the other two. The three test images (all of them being 8 bit grayscale images of size  $512 \times 512$  pixels displayed at 90 dpi, the viewing distance was 0.4 m) were tested at two different compression rates ( $c = 0.25$  bits/pixel

**Table 4** The *PSNR* (*dB*) values  $Q_P$  (compression quality) and  $Q_{PP}$  (artifact reduction) for the maximal artifact reduction. For PAD and NLID,  $K$  is the average gradient magnitude of the original image.

		0.25 bits/pixel			0.4 bits/pixel		
		Boat	Bridge	Goldhill	Boat	Bridge	Goldhill
JPEG2000		31.87	24.88	30.52	34.45	26.31	32.25
JPEG ( $Q_0$ )		30.11	24.09	29.23	32.40	25.39	30.87
$Q_P(t_2)$	NLID	30.14	24.12	29.26	32.43	25.41	30.91
	PAD	30.13	24.12	29.25	32.43	25.46	30.92
	MCMD	30.19	24.22	29.32	32.53	25.49	30.87
$Q_{PP}(t_2)$	NLID	32.02	27.80	32.40	33.95	29.29	33.53
	PAD	31.83	27.41	31.63	33.76	28.15	32.82
	MCMD	32.22	27.94	31.89	33.52	28.09	33.21

and  $c = 0.4$  bits/pixel) with preprocessing up to the maximal artifact reduction. In SE3 the three images were aligned horizontally with the original one in the middle, and it took 30-40 secs to make a choice in average. For SE2 it took 10-15 secs in average.

Since there are a large number of comparisons to be done by a test person, we had to reduce the number of the different diffusion methods involved. LD was less likely to give a large artifact reduction (Fig. 5), it gave poor contour validity (Fig. 7), and our previous results for SE2 (Fig. 9 [17]) have shown that LD always fails against JPEG and any other preprocessing methods when doing maximal artifact reduction. Note that the quality order of different diffusions in Figure 9 is obvious (PAD > NLID > LD), but the strong diffusion caused their failing against JPEG. In a preliminary experiment for SE3, we tested diffusions of the six different combinations of the parameters  $K$  and  $\alpha$  shown in Table 3, then we have chosen the diffusions and parameter settings of the best performance, namely, PAD<sub>1</sub>, NLID<sub>1</sub>, and MCMD. These three methods were compared to JPEG without preprocessing and to JPEG2000 compression in the SE3 and SE2 testing conditions. The images involved in the experiment for “Bridge” are shown in Fig. 10.

## 6.2 Subjective rating of the compression methods

The scales obtained for the SE3 and the SE2 (without JPEG2000) are shown in Fig. 11. We also indicate the number of test persons ( $n$ ) involved in computing the scale and the results of the  $\chi^2$ -test of Mosteller ( $P(\chi^2)$  is the probability that the scale is correct). The *PSNR* values for all images involved are shown in Table 4 for reference.



**Figure 10** JPEG with maximal artifact reduction for “Bridge”, 0.25 bits/pixel. The PSNR values are given in Table 4.

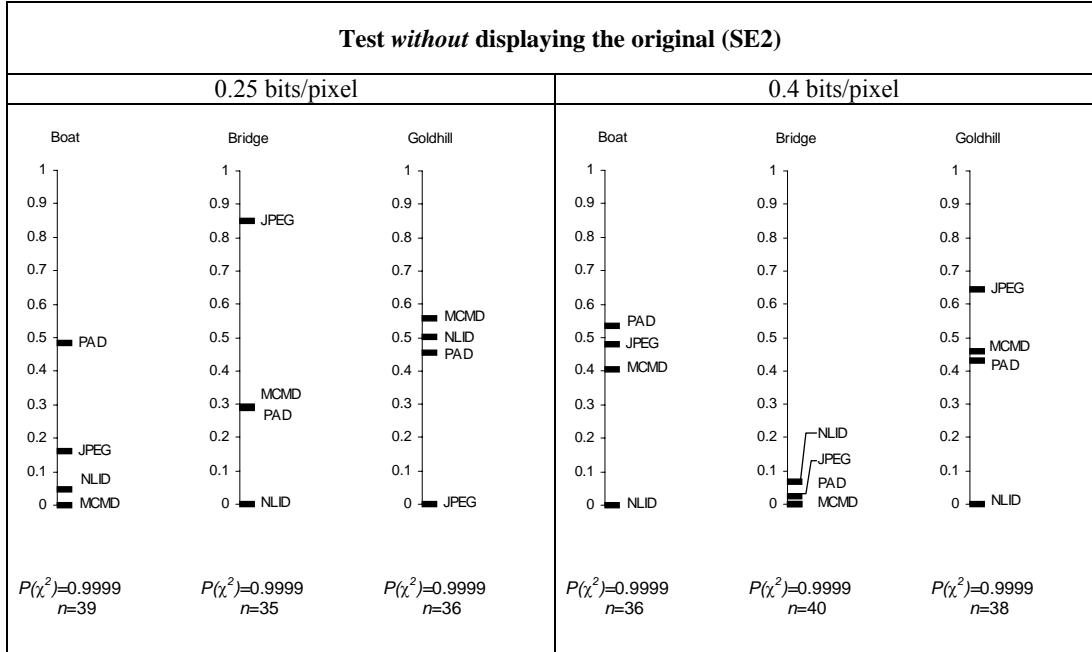
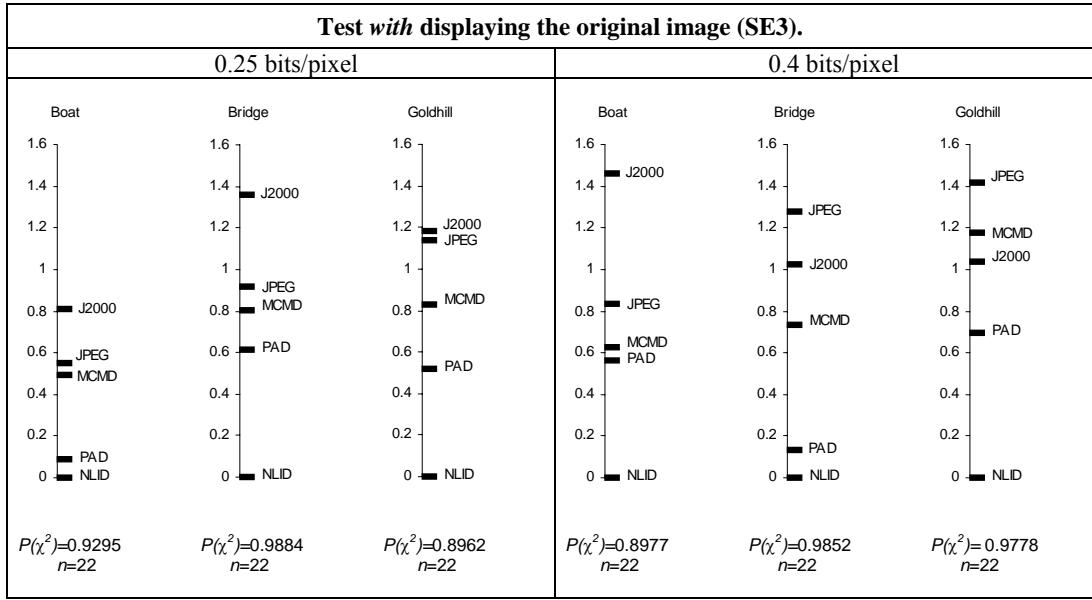
As to SE3, though the compared JPEG compressed images are of almost the same PSNR value (quality values  $Q_P(t_2)$  in Table 4), and though JPEG2000 has a much better PSNR, their preferences by the test persons are quite different. Since the original image is displayed for reference, the test persons could identify details that have been lost, the blur, and possibly slight contrast changes easily. We conclude that if the subjective fidelity to the original is of concern, the test persons perceive the lack of details in the preprocessed images and sometimes in the images compressed with JPEG2000 more disturbing than the artifacts in the JPEG compression without preprocessing.

In most of the applications the original image is not available and the subjective impression of quality is more important than the fidelity to the original one. The SE2 is intended to rate the compression methods concerning this criterion. Though JPEG2000 was involved in all of the measurements, it has proved to be an outlier. On the constructed scale, it has much better values than any other method in either case. This is presumably due to the blocking artifact of JPEG. To eliminate the bias introduced by JPEG2000, we decided to exclude it from the final scale construction.

Another problem with SE2 was that in the lack of the original image as a reference, the test persons could probably apply multiple preference criteria in their choices between the different JPEG compressed versions, which seemed quite similar for a non-expert person. Also, the original scale construction procedure of Thurston assumes “fair” test persons, and does not provide a procedure to check if they made just random choices. Due to this, we had to involve a much larger number of test persons (62) than in SE3, and they were filtered by a simple consistency check. We excluded those measurement data from the computation where the relation “better than” was not transitive. Practically, it does not change the output scale, but it increased the confidence up to nearly 100%.

The conclusion drawn from the scales obtained for SE2 is that if the visual impression of the quality is important, at least for lower bit rates, preprocessing can induce an improvement. When preprocessing increases the validity of contours (“Goldhill” and “Boat”), then a significant majority of the test persons (60-65%) recognizes the improvement with a high confidence (99.99%).

Consequently, the preprocessed images will be compressed with better PSNR quality (i.e., with larger artifact reduction), higher contour validity at low bit rates, and better visual impression of quality.



**Figure 11** Scaling results. Positive distance means “better than”. J2000 = JPEG2000, PAD = PAD<sub>I</sub>, NLID = NLID<sub>I</sub> (Table 3).  $P(\chi^2)$  is the probability that the scale is correct, and  $n$  is the number of the test persons.

The differences in the scales obtained by the two different methods (SE2 and SE3) can be explained as follows. Some areas may contain false edges in case of no preprocessing, but the brain – based on its high adaptability and learning capabilities, and due to some masking effects – may detect contours from the degraded details [13,14]. The preprocessed images can therefore be assessed as worse in the presence of the original due to the lack of artifacts.

Since SE2 is relevant for compression applications, we can conclude from the above experiments that directional adaptability gains its importance if visual impression of quality is of concern. PAD and MCMD are therefore usually better to use than isotropic diffusion. If the image contains high contrast and well structured edges (e.g., the “Boat” image) then PAD is likely to be better than MCMD, since it is also contrast adaptive. For images with more homogenous gray-scale distribution and less sharp edges (like “Goldhill” and “Bridge”), the loss of sharpness is less perceptible and MCMD can achieve nearly the same effect as PAD with a smaller number of iterations. For the same reason, NLID can also achieve similar results as it happens for “Goldhill”.

## 7 Conclusion

We have considered the application of different diffusion methods as a preprocessing step for artifact reduction for block-DCT compression, which we had previously proposed in [34]. A selected class of diffusion processes, potentially useful for artifact reduction, is considered. These can be efficiently implemented [40] and facilitate fast pixel-based parallel hardware processing [30].

The maximal improvement in PSNR achievable with preprocessing was 0.1 to 0.4 dB for our test images, so we decided to allow the largest possible preprocessing not yielding a worse PSNR value compared to the original image as the compression without preprocessing. As a result, the preprocessed image is compressed with 2 to 6 dB better PSNR than the original image. By doing so, we try to preserve the information relevant to image processing applications and visual perception.

Since many image processing algorithms rely on the edge-content of the image and are disturbed by noise, we demonstrated that edge detection on the compressed images is more stable by using diffusion preprocessing. In machine vision applications, compressed images can be better recognized if contours are rather true than false, which can be achieved by diffusion preprocessing.

The hypothesis that the preprocessed images have less visual artifacts was tested by subjective measurements. We obtained that this can be true at lower bit rates. As a side effect, we have also shown that JPEG2000 is better than JPEG or JPEG with preprocessing.

The diffusion models could be improved by better adaptation at the textured regions. This would also be necessary when extending preprocessing to other highly optimized compression methods such as JPEG2000 using spatially adaptive quantization. Our experience is that it is difficult to adopt and evaluate the preprocessing with more complex compression methods like JPEG2000 due to the lack of standard and widely accepted visual quality measures (we can sometimes have a better visual quality with a worse PSNR).

Our presented method could be considered in digital photography, for Motion-JPEG, or extended for video compression. The compressed images could be used for magnification or enhancement (contour searching), where artifacts should be avoided. Our proposal for using non-linear diffusion preprocessing may give a compromise among human perception, preserving of structures, different artifacts/distortions and computing efforts dedicated for their reduction.

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## Appendix

Here we explain the notations used in the description of diffusion equations. For functions  $f : \Re^2 \rightarrow \Re$ , the gradient and the Hessian at  $\mathbf{x} \in \Re^2$  are denoted by  $Df(\mathbf{x})$  respectively  $D^2 f(\mathbf{x})$ . For a function  $u : [0, \infty) \times \Re^2 \rightarrow \Re$  and for fixed  $s > 0$  and  $\mathbf{y} \in \Re^2$  we define

- $u_s : \Re^2 \rightarrow \Re$ ,  $u_s(\mathbf{x}) = u(s, \mathbf{x})$ ,  $\mathbf{x} \in \Re^2$ ,
- $u_{\mathbf{y}} : [0, \infty) \rightarrow \Re$ ,  $u_{\mathbf{y}}(t) = u(t, \mathbf{y})$ ,  $t > 0$ .

For  $\mathbf{v} = (v_1, v_2) \in \Re^2$ , let  $\mathbf{v}^\perp = (-v_2, v_1)$ . We have then the following notations:

<i>Time derivative</i>	$\partial_t u(s, \mathbf{x}) = u'_{\mathbf{x}}(s)$	<i>Laplacian operator</i>	$\Delta u = \text{trace}(\nabla^2 u)$
<i>Spatial gradient</i>	$\nabla u(t, \mathbf{x}) = Du_t(\mathbf{x})$	<i>Edge-parallel component</i>	$u_{\parallel} = (\nabla u^{\perp} /  \nabla u )^T \nabla^2 u (\nabla u^{\perp} /  \nabla u )$
<i>Spatial Hessian matrix</i>	$\nabla^2 u(t, \mathbf{x}) = D^2 u_t(\mathbf{x})$	<i>Edge-normal component</i>	$u_{\perp} = (\nabla u /  \nabla u )^T \nabla^2 u (\nabla u /  \nabla u )$

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