SOME RESULTS ON THE DERIVED SERIES OF FINITE p-GROUPS THESIS ABSTRACT

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It is well-known that in a finite p-group G, the condition $|G'/G''| \leqslant p^2$ implies that G' is abelian. More generally, if $|G^{(k)}/G^{(k+1)}| \leqslant p^{2^k}$ for some $k \geqslant 1$, then $G^{(k)}$ is abelian. The main objective of this dissertation is to report on finite p-groups G, such that $|G^{(k)}/G^{(k+1)}| = p^{2^k+1}$ and $G^{(k+1)} \neq 1$ for some $k \geqslant 1$.

First we study this condition for $k = 1$; that is we assume that $|G'/G''| = p^3$ and $G'' \neq 1$. It is a result of Hall that for odd primes $|G''| = p$, and so $|G'| = p^4$. We improve Hall's result by showing that $\gamma_3(G)$ is a maximal subgroup of G' and $G'' = \gamma_5(G)$. Moreover G' is the direct product of an extraspecial group of order p^3 and a cyclic group of order p. If p is even then the order of G'' can be arbitrarily large. In this case Blackburn showed that G'' is abelian with at most two generators. The set of such 2-groups can be divided into two classes according to whether $\gamma_3(G)$ is a maximal subgroup of G' or not, and we show that neither class is empty. Furthermore, we refine Blackburn's results by obtaining a more detailed description of the quotients $\gamma_i(G)/\gamma_{i+1}(G)$ for $i \geqslant 2$.

It is an undecided question whether for $k \geqslant 2$ and $p \geqslant 3$ there exists a finite p-group G, such that $|G^{(k)}/G^{(k+1)}| = p^{2^k+1}$ and $G^{(k+1)} \neq 1$. We find that if such G exists, then $[G^{(k)}, G]$ must be a maximal subgroup in $G^{(k)}$. For $p \geqslant 5$ we also show that $G^{(k)}$ has nilpotency class at most 3, and its order is bounded by a function of the form $p^{f(k)}$, where f is independent of p. These results are based on Bokut's similar results on Lie algebras.

Our discussion leads to a new lower bound for the order of a p-group with a given soluble length. In particular, we prove that for $p \neq 3$ the condition $G^{(k)} \neq 1$ implies $|G| \geqslant p^{2^k+3k-8}$. We describe some examples for groups and Lie algebras of high soluble length and low order. Among others, we present constructions published elsewhere by the author, M. F. Newman, and S. Evans-Riley for p-groups with soluble length k and order p^{2^k-2}. These groups are currently the smallest known.

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Most of our theorems are proved using Lie methods, and some interesting results about Lie algebras are also obtained.

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