Bounds on $K_6(n, R)$

(lower and upper bounds on the size of senary optimal covering codes)

n	$R=1$	$R=2$	$R=3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{6}^{11}$	$\mathbf{1}^{1}$	
3	g 18 g	6	$\mathbf{1}^{1}$
4	i 72 i	k 15 n	6
5	r 330–414 l	m 36–66 n	k 12 q
6	r 1578–1840 l	m 133–274 q	m 24–41 q
7	q 7777–11040 e	y 528–1296 f	m 70–246 e
8	y 41991–62208 j	y 2276–5184 f	m 246–1080 f
9	u 219096–324864 j	y 10900–29808 f	y 921–4752 f
10	y 1209324–1866240 j	y 53463–132480 f	y 3815–19347 k

n	$R=4$	$R=5$	$R=6$	$R=7$	$R=8$
4	$\mathbf{1}^{1}$				
5	6	$\mathbf{1}^{1}$			
6	k 10 n	6	$\mathbf{1}^{1}$		
7	m 18–36 n	$\mathbf{6}^{1}$	6	$\mathbf{1}^{1}$	
8	m 46–216 e	m 15–30 z	$\mathbf{6}^{11}$	6	$\mathbf{1}^{1}$
9	m 136–738 f	m 32–144 j	k 12–22 q	6	6
10	s 417–2952 f	m 83–615 f	m 25–72 n	k 12–18 r	6

Bounds on $K_7(n, R)$

(lower and upper bounds on the size of septenary optimal covering codes)

n	$R=1$	$R=2$	$R=3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{7}^{15}$	$\mathbf{1}^{1}$	
3	g 25 g	7	$\mathbf{1}^{1}$
4	b 115–123 l	b 17–19 n	7
5	r 606–769 l	m 55–97 n	k 15–17 q
6	r 3412–4435 l	m 233–343 p	m 36–77 q
7	r 19818–31045 e	x 1035–2401 e	m 127–343 c
8	h 117649 h	x 5457–15129 f	y 457–2337 f
9	u 733726–823543 e	s 29889–94587 f	y 2077–8575 f
10	u 4630843–5764801 e	x 168042–420175 q	y 10577–42189 f

n	$R=4$	$R=5$	$R=6$	$R=7$	$R=8$
4	$\mathbf{1}^{1}$				
5	7	$\mathbf{1}^{1}$			
6	k 13–15 n	7	$\mathbf{1}^{1}$		
7	m 25–49 p	k 11 n	7	$\mathbf{1}^{1}$	
8	m 76–343 c	m 20–49 c	$\mathbf{7}^{1}$	7	$\mathbf{1}^{1}$
9	m 264–1843 f	m 52–323 f	m 17–37 n	$\mathbf{7}^{15}$	7
10	y 1007–6517 f	m 160–1225 f	m 39–175 o	k 15–33 z	7

Bounds on $K_8(n, R)$
(lower and upper bounds on the size of octonary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^1$		
2	$\mathbf{8}^{22}$	$\mathbf{1}^1$	
3	g 32 g	8	$\mathbf{1}^1$
4	b 171–192 l	b 22–23 n	8
5	i 1024 i	m 83–128 n	b 17–22 q
6	r 6626–8192 e	m 382–512 p	m 52–107 q
7	r 44237–63488 j	y 1984–4096 e	m 196–512 c
8	r 302036–342272 l	y 11766–29920 j	y 829–4096 c
9	h 2097152 h	y 75783–174080 j	y 4523–16384 f
10	y 15339169–16777216 e	y 478586–1048576 f	y 25767–98304 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^1$				
5	8	$\mathbf{1}^1$			
6	k 15–20 n	8	$\mathbf{1}^1$		
7	m 37–92 n	k 14–16 n	8	$\mathbf{1}^1$	
8	m 118–512 c	m 29–90 z	k 12 n	8	$\mathbf{1}^1$
9	s 409–2944 f	m 80–384 j	m 22–48 n	$\mathbf{8}^1$	8
10	y 2045–11776 f	m 287–2461 f	m 58–342 n	m 20–44 z	$\mathbf{8}^{22}$

Bounds on $K_9(n, R)$
(lower and upper bounds on the size of novenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^1$		
2	$\mathbf{9}^{30}$	$\mathbf{1}^1$	
3	g 41 g	9	$\mathbf{1}^1$
4	i 243 i	b 27 n	9
5	b 1641–1809 l	m 113–189 n	b 21–27 c
6	r 11877–16010 l	m 585-729 p	m 71–147 q
7	r 89526–135594 j	y 3536–6561 e	m 308–729 c
8	r 690377–875259 l	x 23184–59049 e	x 1413–6561 c
9	r 5415428–6053198 l	y 165310–439587 f	y 8544–29889 f
10	h 43046721 h	y 1206918–3272481 f	x 54144–177147 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^1$				
5	9	$\mathbf{1}^1$			
6	k 18–24 q	9	$\mathbf{1}^1$		
7	m 51–120 n	k 16–21 n	9	$\mathbf{1}^1$	
8	m 181–729 c	m 39–120 c	k 15–17 n	9	$\mathbf{1}^1$
9	s 703–5103 f	m 120–729 c	m 31–81 q	k 13 n	9
10	y 3872–19683 f	m 481–3969 f	m 87–477 n	m 25–77 z	$\mathbf{9}^1$

Bounds on $K_{10}(n, R)$
(lower and upper bounds on the size of denary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{10}^{42}$	$\mathbf{1}^{1}$	
3	g 50 g	10	$\mathbf{1}^{1}$
4	b 334–352 l	b 34 n	10
5	b 2501–2944 l	m 149–250 n	b 26–32 q
6	i 20000 i	m 890–1350 q	m 92–209 q
7	r 167925–200000 e	y 5676–13500 e	m 451–1350 c
8	r 1442623–2000000 e	y 42772–123904 f	y 2415–11968 f
9	r 12608696–15614398 l	x 333560–1000000 f	y 15959–67500 f
10	r 111688312–123456288 l	a 2676660–7040000 f	y 110867–475200 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	10	$\mathbf{1}^{1}$			
6	b 21–28 q	10	$\mathbf{1}^{1}$		
7	m 66–168 n	k 19–26 n	10	$\mathbf{1}^{1}$	
8	m 265–1156 f	m 48–168 c	k 17–22 n	10	$\mathbf{1}^{1}$
9	s 1130–8500 f	m 174–1088 f	m 39–114 q	k 16-18 n	10
10	y 6886–45900 f	y 632–7106 f	m 122–826 n	m 33–110 z	k 14 n

Bounds on $K_{11}(n, R)$
(lower and upper bounds on the size of undenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{11}^{56}$	$\mathbf{1}^{1}$	
3	g 61 g	11	$\mathbf{1}^{1}$
4	b 444–485 l	b 41 n	11
5	b 3661–4309 l	m 208–365 q	b 31–38 q
6	b 32211–35315 l	m 1276–2343 q	m 119–275 q
7	r 296388–388465 e	y 9106–14641 p	m 636–2343 c
8	r 2806050–4198148 l	s 74415–161051 e	y 3732–14641 c

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	11	$\mathbf{1}^{1}$			
6	b 25–33 q	11	$\mathbf{1}^{1}$		
7	m 84–216 n	k 22–31 n	11	$\mathbf{1}^{1}$	
8	m 374–1681 f	m 64–216 c	k 20–27 n	11	$\mathbf{1}^{1}$

Bounds on $K_{12}(n, R)$
(lower and upper bounds on the size of duodenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{12}^{77}$	$\mathbf{1}^{1}$	
3	g 72 g	12	$\mathbf{1}^{1}$
4	i 576 i	b 48 n	12
5	i 5184 i	m 256–468 n	b 37–43 z
6	b 49767–58880 l	m 1764–3884 q	m 144–384 j
7	b 497664–706560 e	y 14036–26920 k	m 878–3456 f
8	r 5146894–7558272 j	y 123772–323040 e	y 5577–26920 c

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	12	$\mathbf{1}^{1}$			
6	k 30–41 q	12	$\mathbf{1}^{1}$		
7	m 104–264 n	b 25–36 n	12	$\mathbf{1}^{1}$	
8	m 513–2304 f	m 78–264 c	k 23–32 n	12	$\mathbf{1}^{1}$

Bounds on $K_{13}(n, R)$
(lower and upper bounds on the size of tredenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{13}^{101}$	$\mathbf{1}^{1}$	
3	g 85 g	13	$\mathbf{1}^{1}$
4	b 733–765 l	b 57 n	13
5	b 7141–7654 l	m 311–583 q	b 43–48 q
6	b 74259–92535 l	x 2169–6127 q	m 193–495 q
7	b 804469–1202955 e	x 20189–47111 q	m 1198–4294 n
8	r 8988848–12259982 l	x 197563–371293 p	x 8086–28561 p

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	13	$\mathbf{1}^{1}$			
6	k 35–46 q	13	$\mathbf{1}^{1}$		
7	m 132–356 q	b 29–42 n	13	$\mathbf{1}^{1}$	
8	m 684–3249 f	m 98–356 c	b 25–37 n	13	$\mathbf{1}^{1}$

Bounds on $K_{14}(n, R)$
(lower and upper bounds on the size of quattuordenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{14}^{135}$	$\mathbf{1}^{1}$	
3	g 98 g	14	$\mathbf{1}^{1}$
4	b 915–984 l	b 66 n	14
5	b 9605–10858 l	m 381–686 n	b 50–54 n
6	b 107565–130665 l	y 2955–7203 j	m 235–610 q
7	b 1254923–1829310 e	y 29273–78934 q	m 1570–4802 n
8	i 15059072 i	y 305294–671728 k	y 11878–51670 k

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	14	$\mathbf{1}^{1}$			
6	b 40–52 q	14	$\mathbf{1}^{1}$		
7	m 162–448 q	k 34–48 n	14	$\mathbf{1}^{1}$	
8	m 917–4356 f	m 119–448 c	b 29–42 n	14	$\mathbf{1}^{1}$

Bounds on $K_{15}(n, R)$
(lower and upper bounds on the size of quindenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{15}^{176}$	$\mathbf{1}^{1}$	
3	g 113 g	15	$\mathbf{1}^{1}$
4	i 1125 i	b 75 n	15
5	b 12657–14289 l	m 465–855 n	b 57–59 n
6	i 151875 i	y 3812–10625 j	m 291–730 q
7	b 1898438–2278125 e	y 41361–127125 f	s 1745–6497 n
8	b 24408483–26150859 l	x 457584–1166533 k	y 16380–84375 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	15	$\mathbf{1}^{1}$			
6	b 46–59 c	15	$\mathbf{1}^{1}$		
7	m 192–519 n	k 39–54 n	15	$\mathbf{1}^{1}$	
8	m 1181–5625 f	m 141–519 c	b 33–49 n	15	$\mathbf{1}^{1}$

Bounds on $K_{16}(n, R)$
(lower and upper bounds on the size of sexdenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{16}^{231}$	$\mathbf{1}^{1}$	
3	g 128 g	16	$\mathbf{1}^{1}$
4	b 1366–1416 l	b 86–87 n	16
5	i 16384 i	m 576–1024 n	b 64 n
6	b 209716–223360 l	x 4848–10752 j	m 344–896 j
7	b 2796203–3573760 e	y 55600–172032 e	s 2226–8192 n
8	b 38347923–43810816 l	y 669207–1954905 k	y 22167–123192 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	16	$\mathbf{1}^{1}$			
6	b 52–64 c	16	$\mathbf{1}^{1}$		
7	m 227–611 q	k 44–60 n	16	$\mathbf{1}^{1}$	
8	m 1507–7569 f	m 168–611 c	k 38–56 n	16	$\mathbf{1}^{1}$

Bounds on $K_{17}(n, R)$
(lower and upper bounds on the size of septemdenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{17}^{297}$	$\mathbf{1}^{1}$	
3	g 145 g	17	$\mathbf{1}^{1}$
4	b 1638–1743 l	b 97–99 n	17
5	b 20881–22033 l	m 671–1241 n	b 73 n
6	b 283972–319925 l	y 6260–14424 q	m 407–1241 e
7	b 4022929–5438725 e	y 75429–252735 f	s 2806–10657 n
8	b 58619811–70994315 l	x 955978–3038049 f	x 29478–172557 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	17	$\mathbf{1}^{1}$			
6	k 59–73 c	17	$\mathbf{1}^{1}$		
7	m 271–703 q	b 49–66 n	17	$\mathbf{1}^{1}$	
8	y 1458–9801 f	m 198–703 c	k 43–63 n	17	$\mathbf{1}^{1}$

Bounds on $K_{18}(n, R)$
(lower and upper bounds on the size of octodenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{18}^{385}$	$\mathbf{1}^{1}$	
3	g 162 g	18	$\mathbf{1}^{1}$
4	i 1944 i	b 109–111 n	18
5	b 26245–28904 l	m 807–1458 n	b 82 n
6	b 377914–442240 l	y 7741–15309 j	m 471–1353 j
7	b 5668704–7960320 e	y 100579–275562 e	s 3492–13122 n
8	b 87460005–110939643 l	y 1339650–3779136 f	y 39755–215784 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	18	$\mathbf{1}^{1}$			
6	k 66–80 n	18	$\mathbf{1}^{1}$		
7	m 316–774 q	b 55–72 n	18	$\mathbf{1}^{1}$	
8	s 1839–12321 f	m 233–774 c	k 48–70 n	18	$\mathbf{1}^{1}$

Bounds on $K_{19}(n, R)$
(lower and upper bounds on the size of undevicenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^{1}$		
2	$\mathbf{19}^{490}$	$\mathbf{1}^{1}$	
3	g 181 g	19	$\mathbf{1}^{1}$
4	b 2287-2359 l	b 121–123 n	19
5	b 32581–35881 l	m 957–1779 y	b 91 n
6	b 495220–571300 l	x 9479–20890 q	m 557–1568 q
7	b 7840981–10854700 e	x 128972–396910 e	x 4237–18737 n
8	b 127695963–164570091 l	x 1842639–5564881 f	y 51278–290157 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^{1}$				
5	19	$\mathbf{1}^{1}$			
6	b 73–86 n	19	$\mathbf{1}^{1}$		
7	m 367–866 q	b 61–81 n	19	$\mathbf{1}^{1}$	
8	y 2256–15129 f	m 271–866 c	k 53–77 n	19	$\mathbf{1}^{1}$

Bounds on $K_{20}(n, R)$
(lower and upper bounds on the size of vicenary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^1$		
2	$\mathbf{20}^{627}$	$\mathbf{1}^1$	
3	g 200 g	20	$\mathbf{1}^1$
4	b 2667-2816 l	b 134–135 n	20
5	i 40000 i	m 1088–2000 j	b 100 n
6	i 640000 i	y 11778–28160 q	m 650–1600 j
7	b 10666667–12800000 e	y 167165–563200 f	x 5174–21202 n
8	b 182857143–223741497 l	y 2495614–7929856 f	y 65351–380160 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^1$				
5	20	$\mathbf{1}^1$			
6	b 81–93 n	20	$\mathbf{1}^1$		
7	m 419–979 q	k 68–89 n	20	$\mathbf{1}^1$	
8	y 2760–18225 f	m 312–979 c	b 58–84 n	20	$\mathbf{1}^1$

Bounds on $K_{21}(n, R)$
(lower and upper bounds on the size of viginti-unary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^1$		
2	$\mathbf{21}^{792}$	$\mathbf{1}^1$	
3	g 221 g	21	$\mathbf{1}^1$
4	i 3087 i	b 147 n	21
5	b 48621–50796 l	m 1257–2381 y	b 111–114 n
6	b 816821–856885 l	y 14131–37481 q	m 738–2058 j
7	b 14294354–17994585 e	y 213881–571438 j	x 6249–28812 j
8	i 257298363 i	x 3329193–9529569 f	x 82341–453789 f

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^1$				
5	21	$\mathbf{1}^1$			
6	b 89–99 n	21	$\mathbf{1}^1$		
7	m 497–1029 n	k 75–98 n	21	$\mathbf{1}^1$	
8	s 3288–21609 e	m 354–1029 c	b 64–91 n	21	$\mathbf{1}^1$

Key to the tables for $K_q(n, R)$, $q \geq 6$

	Lower bounds
unmarked	trivial
a	sphere-covering bound
b	(Rodemich, 1970)
g	(Kalbfleisch–Stanton, 1969)
h	Hamming code
i	(Blokhuis–Lam, 1984)
k	(Haas–Schlage-Puchta–Quistorff, 2008–2009)
m	(Haas–Halupczok–Schlage-Puchta, 2009)
q	non-existence of perfect code
r	(Quistorff, 2001)
s	(Lang–Quistorff–Schneider, 2006–2007)
u	(Haas, 2000)
x	(Chen–Honkala, 1990)
y	improved sphere-covering bound

	Upper bounds
unmarked	trivial
c	$K_q(n+1, R+1) \leq K_q(n, R)$
e	$K_q(n+1, R) \leq qK_q(n, R)$
f	direct sum
g	(Kalbfleisch–Stanton, 1969)
h	Hamming code
i	(Blokhuis–Lam, 1984)
j	$K_{tq}(n, 1) \leq t^{n-1}K_q(n, 1)$ or $K_{tq}(n, R) \leq \sigma_t(n, n-R)K_q(n, R)$
k	using known bound for the same n, R and smaller q
l	constructions using adjoint codes
n	(Kéri–Östergård, 2005)
o	(Östergård, 1999)
p	linear code
q	(Rivas Soriano, 2005–2008)
r	(Quistorff–Schlage-Puchta, 2008)
y	(Li–Ji-Yin, 2008)
z	(Kéri, 2009)