Bounds on $K_4(n, R)$
(lower and upper bounds on the size of quaternary optimal covering codes)

n	$R = 1$	$R = 2$	$R = 3$
1	$\mathbf{1}^1$		
2	$\mathbf{4}^5$	$\mathbf{1}^1$	
3	g $\mathbf{8}^1$ g	$\mathbf{4}^{21}$	$\mathbf{1}^1$
4	g 24 z	i $\mathbf{7}^8$ n	$\mathbf{4}^{79}$
5	h 64 h	k 16 m	$\mathbf{4}^1$
6	y 228–256	j 32–52 o	k 11–14 x
7	z 762–992 x	m 84–128 o	m 19–32 m
8	y 2731–3456 o	y 240–352 x	m 44–96 m
9	x 9368–12288 o	x 751–1024 o	m 110–256 m
10	y 34953–49152	z 2412–4096	y 313–832 x
11	z 123846–131072 w	y 7974–15872 x	y 849–2048 x

n	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
4	$\mathbf{1}^1$				
5	$\mathbf{4}^{269}$	$\mathbf{1}^1$			
6	$\mathbf{4}^5$	$\mathbf{4}^{839}$	$\mathbf{1}^1$		
7	k 9–10 q	$\mathbf{4}^{21}$	4	$\mathbf{1}^1$	
8	w 13–28 m	t 8 m	$\mathbf{4}^{79}$	4	$\mathbf{1}^1$
9	m 26–64 m	m 10–16 m	$\mathbf{4}^1$	$\mathbf{4}^{269}$	4
10	m 59–208 o	m 18–54 q	t 8–16	$\mathbf{4}^5$	$\mathbf{4}^{839}$
11	y 148–512 v	m 36–128 v	m 14–32 v	t 8–11 q	$\mathbf{4}^{21}$

Bounds on $K_5(n, R)$
(lower and upper bounds on the size of quinary optimal covering codes)

n	$R=1$	$R=2$	$R=3$
1	$\mathbf{1}^1$		
2	$\mathbf{5}^7$	$\mathbf{1}^1$	
3	g $\mathbf{13}^1$ g	$\mathbf{5}^{54}$	$\mathbf{1}^1$
4	s 46–51 z	k 11 n	$\mathbf{5}^{471}$
5	u 160–184 z	m 22–35 m	k 9 n
6	h 625 h	m 71–125 m	m 16–25 m
7	z 2722–3125	m 225–525 o	m 38–100 u
8	z 11945–15625	x 821–1625 q	m 109–325 d
9	z 53138–78125	y 3367–6375 q	p 330–1275 d
10	z 238993–390625	y 13161–23000 q	x 1163–3125 p
11	y 1087416–1525877 k	z 52842–78125 q	f 4255–15625 q

n	$R=4$	$R=5$	$R=6$	$R=7$	$R=8$
4	$\mathbf{1}^1$				
5	5	$\mathbf{1}^1$			
6	$\mathbf{5}^1$	5	$\mathbf{1}^1$		
7	m 12–21 q	$\mathbf{5}^7$	5	$\mathbf{1}^1$	
8	m 25–65 d	k 11-15 q	$\mathbf{5}^{54}$	5	$\mathbf{1}^1$
9	m 64–255 d	m 19–55 d	k 10–12 q	$\mathbf{5}^{471}$	5
10	y 162–875 d	m 41–175 d	m 16–45 d	k 9 y	5
11	y 535–3125 d	m 103–625 d	m 29–125 d	m 12–25 d	$\mathbf{5}^1$

Key to the tables for $K_4(n, R)$, $K_5(n, R)$

	Lower bounds
unmarked	trivial
f	(Chen–Honkala, 1990)
g	(Kalbfleisch–Stanton, 1969)
h	Hamming code
i	(Cohen–Honkala–Litsyn–Lobstein, 1997)
j	(Haas, 2006)
k	(Haas–Schlage-Puchta–Quistorff, 2008–2009)
m	(Haas–Halupczok–Schlage-Puchta, 2009)
p	(Habsieger–Plagne, 2000)
s	(Kamps–van Lint, 1970)
t	(Kéri–Östergård, 2003)
u	(Quistorff, 2001)
w	(Kéri, 2008)
x	(Lang–Quistorff–Schneider, 2006)
y	improved sphere-covering bound
z	(Gijswijt, 2005)

	Upper bounds
unmarked	trivial
d	(Bhandari–Durairajan, 1996)
g	(Kalbfleisch–Stanton, 1969)
h	Hamming code
k	using known bound for the same n, R and smaller q
m	(Östergård, 1991)
n	(Östergård, 1994)
o	(Östergård, 1999)
p	(Davydov–Östergård, 2000)
q	(Rivas Soriano, 2006–2008)
u	(Gommard–Plagne, 2003)
v	amalgamated direct sum
w	(Välinen, 2002)
x	(Bertolo–Di Pasquale–Santisi, 2004–2006)
y	(Mendes–Monte Carmelo–Poggi, 2009)
z	(Stanton–Horton–Kalbfleisch, 1969)