

# Joint optimization of product tolerance design, process plan, and production plan in high-precision multi-product assembly

Daisuke Tsutsumi<sup>\*a,b</sup>, Dávid Gyulai<sup>c</sup>, András Kovács<sup>c</sup>, Bence Tipary<sup>c</sup>,  
Yumiko Ueno<sup>a</sup>, Youichi Nonaka<sup>a</sup>, Kikuo Fujita<sup>b</sup>

<sup>a</sup>*Hitachi Ltd., Research & Development Group, 292 Yoshidacyo, Totsuka, Yokohama, Kanagawa, 244-0817, Japan*

<sup>b</sup>*Department of Mechanical Engineering, Osaka University, 2-1 Yamadaoka, Suita, Osaka 565-0871, Japan*

<sup>c</sup>*EPIC Centre of Excellence in Production Informatics and Control, Institute for Computer Science and Control (SZTAKI), Kende u. 13-17, H-1111 Budapest, Hungary*

---

## Abstract

With the ever-increasing product variety faced by the manufacturing industry, investment efficiency can only be maintained by the application of multi-product assembly systems. In such systems, the product design, process planning, and production planning problems related to different products are strongly interconnected. Despite this, those interdependent decisions are typically made by different divisions of the company, by adopting a decomposed planning approach, which can easily result in excess production costs. In order to overcome this challenge, this paper proposes an integrated approach to solving the above problems, focusing on the decisions crucial for achieving the required tolerances in high-precision assembled products. The joint optimization problems related to product tolerance design and assembly resource configuration are first formulated as a mixed-integer linear program (MILP). Then, a large neighborhood search (LNS) algorithm, which combines classical mathematical programming and meta-heuristic techniques, is introduced to solve large instances of the problem. The efficiency of the method is demonstrated through an industrial case study, both in terms of computational efficiency and industrial effectiveness.

---

\*Corresponding author

*Email address:* `daisuke.tsutsumi.jh@hitachi.com` (Daisuke Tsutsumi\*)

*Keywords:* Assembly, Design optimization, Tolerancing, Process planning, Production planning

---

## 1. Introduction and motivation

In response to diversifying consumer preferences, many companies from the automotive, electronics, and consumer goods industries are forced to increase product variety [1, 2, 3]. The situation is often complicated further by the changes of the conventional manufacturer-supplier relationships, e.g., in the automotive industry, where a single supplier now serves many manufacturers. Therefore, the supplier must increase its product variety, and the demand for multi-variety production grows. As a consequence, requirements of new products often cannot be satisfied by existing manufacturing and assembly lines, and therefore, investment into new equipment is inevitable. There are also attempts to lift manufacturing constraints by introducing general purpose equipment, but excessive generalization or flexibility of equipment can also lead to low production rate and low return on investments [4].

In the conventional product development process, different phases of the process focus on different issues to be resolved: first of all, product design has to meet customer specifications by selecting appropriate design alternatives. When a product design is available, process planning is responsible for realizing the design by defining the assembly resource configurations. In the operation stage, production planning assigns products to resources over time to satisfy demand in the most efficient way. An important business challenge is to maintain profits via internal efficiency by minimizing total production costs while using existing assembly resources efficiently. However, with the ever-changing product portfolio, not only the existing resources, but also investments into new production resources are part of the game.

The increase of product variety is often led by the product design department, whereas process and production planning are carried out in subsequent steps [5]. Hence, in the conventional product development process, there is no appropriate feedback mechanism, and as a result of limited consideration of production aspects in product design, it is not possible to benefit from the introduction of a common assembly system that enables multi-product assembly [6]. Consequently, individually optimized and less versatile assembly systems are abused, leading to a decline in return on assets due to excessive investment [7]. In general, the key challenge in multi-product assembly is to

34 find the best tradeoff between product design, process and production plan-  
 35 ning aspects considering a portfolio of diverse products, changing demand  
 36 volumes, alternative resources, and investment options over time.

37 However, there are traditional walls among the product design, process  
 38 planning, and production planning domains that altogether constitute the  
 39 product development process [5, 6]. Several traditional methods, for example  
 40 the well-known *Design for Manufacturing and Assembly* (DFMA) approach  
 41 [8] attempt to break these walls and enforce production aspects in product  
 42 design. Nevertheless, their application is limited, and they often provide un-  
 43 satisfactory feedback. The objective of this research is to open new avenues  
 44 from production back to product design for the efficient use of existing as-  
 45 sembly resources. It is important to highlight that the proposed method is  
 46 completely based on formal mathematical models, instead of the commonly  
 47 applied rule-based decisions.

48 The product development process targeted by this research has many sub-  
 49 processes with complex interdependencies. Among these sub-problems, focus  
 50 is given to product tolerance design and resource configuration for assembly  
 51 processes related to achieving the specified tolerances. As product quality is  
 52 also affected by tolerance schemes [9], tolerance design is also one of the most  
 53 important steps in product design development. Figure 1 shows the location  
 54 of tolerance design and assembly resource configuration within the product  
 55 development process.

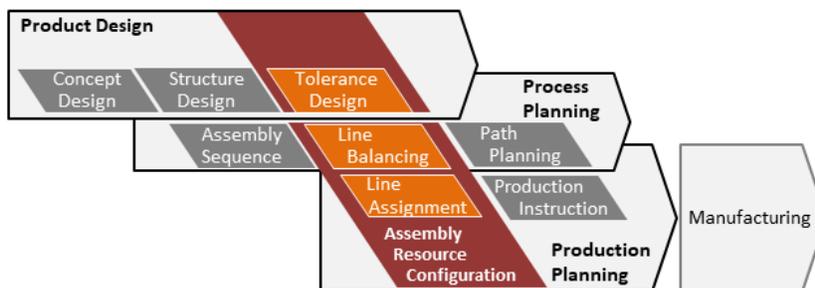


Figure 1: Location of tolerance design and assembly resource configuration within the product development process. Figure adapted from [10].

56 The structure of the paper is as follows. In Section 2, a literature review is  
 57 provided, summarizing conventional approaches in each area of the product  
 58 development process. In Section 3, the problem in scope is formally defined.

59 Section 4 introduces the proposed solution approach in detail. Then, a case  
60 study is provided in Section 5 to evaluate the efficiency of the proposed  
61 methodology in terms of computational efficiency and industrial effectiveness.  
62 An outlook on practical applications is given in Section 6. Finally, conclusions  
63 are drawn and directions for future research are pointed out.

## 64 2. Literature review

65 The product development process targeted in this research can be divided  
66 into product design, which determines the functionality, structure, and ge-  
67 ometry of the product; process planning, which defines the manufacturing  
68 and assembly technologies together with the required resources to produce  
69 the product according to its specifications; and production planning that  
70 matches the manufacturing and assembly process to resources over time to  
71 satisfy demand. This section summarizes the state-of-the-art in each of the  
72 above fields, with special attention to earlier attempts to integrate them.

### 73 2.1. Product design stage

74 While various systematic methodologies have been defined in the liter-  
75 ature to support product design [11, 12], a major step was taken towards  
76 the consideration of subsequent stages of the product development process  
77 with the introduction of various *Design for eXcellence* (DFX) approaches.  
78 Notably, *Design for Manufacturing* (DFM) focuses on the ease of manu-  
79 facturing the individual parts; *Design for Assembly* (DFA) addresses the  
80 efficiency of assembling the parts; whereas *Design for Manufacturing and*  
81 *Assembly* (DFMA) seeks to combine the benefits of both DFM and DFA.  
82 These methods all seek to reduce overhead, material and labor costs, as well  
83 as product development time by using standards and defining appropriate  
84 rules. At the same time, they focus on different stages of the production pro-  
85 cess and accordingly, apply different strategies. The most widespread DFMA  
86 approaches are Boothroyd and Dewhurst’s method [8], the general production  
87 checklists by Huang [13], the Hitachi Assembly Evaluation Method (AEM)  
88 [14], the Lucas method [15] and assembly-oriented design by Redford and  
89 Chal [16]. Design frameworks and automatic tools are proposed to exploit  
90 concurrent design possibilities, considering product life-cycle features already  
91 in the early conceptual design phase. Molcho et al. [17] take on bridging the  
92 gap between the designers, process planners and manufacturers by establish-  
93 ing a knowledge and rule base.

94 Nevertheless, the crucial role of generic guidelines and rules of thumb is  
95 ubiquitous in the above approaches, except for some specific applications,  
96 such as the design of battery systems for electric vehicles in [18]. A ma-  
97 jor problem with conventional DFMA in general multi-product assembly is  
98 that too strict guidelines and the difficulty of updating the guidelines make  
99 it impossible to avoid product designs that violate the guidelines. As a re-  
100 sult, investment into additional manufacturing and assembly equipment is  
101 inevitable, which means that the efficiency of conventional DFMA decreases  
102 in multi-product assembly.

103 While most contributions on product design and process planning deal  
104 with ideal, nominal products, real manufactured and assembled products  
105 never match the nominal design precisely. On the contrary, the allocated  
106 tolerances are decisive on the applicable manufacturing and assembly pro-  
107 cesses, and consequently, on production costs as well. For this reason, this  
108 paper focuses primarily on the tolerance design sub-problem of product de-  
109 sign.

110 In reality, product geometry and dimensions deviate from the nominal  
111 because of variations during both the manufacturing and the assembly pro-  
112 cesses. *Dimensional tolerances* have been for long the primary means for  
113 expressing the allowable deviations of parts and products, and geometrical  
114 tolerances have been formally defined and standardized only recently by the  
115 introduction of *Geometrical Dimensioning and Tolerancing* (GD&T) [19] for  
116 a richer characterization of the allowed deviation. For modeling *cascading*  
117 *tolerances* in assemblies, vector-chain approaches are the most widespread  
118 both in academics and industrial practice [20]. This approach, as well as all  
119 other mainstream models assume that relevant quality features of the final  
120 product are described by so-called *Functional Key Characteristics* (FKCs)  
121 which are influenced by different factors. The dimension chains related to  
122 different FKCs are often interrelated. Despite this, until the 1990s, all ma-  
123 jor works on tolerance optimization assumed independent dimension chains.  
124 The first contribution in tolerance design that can handle interrelated dimen-  
125 sion chains is considered to be [21]. A method for evaluating multiple FKCs  
126 simultaneously in the assembly of compliant parts, such as sheet metal, us-  
127 ing a combination of *Finite Element Analysis* (FEA), tolerance analysis, and  
128 Monte Carlo simulation is proposed in [22].

129 Product tolerance design, when product structure is perfectly defined, re-  
130 duces to the problem of *tolerance allocation*, i.e., assigning tolerances to given  
131 individual dimensions. Various computational approaches have been applied

132 to solving this problem, including genetic algorithms [21, 23], ant colony op-  
133 timization [24], particle swarm optimization [20], numerical methods [25, 26]  
134 for optimizing a well-defined objective, as well as ontologies and rules to  
135 determine tolerances by exploiting technological knowledge (but without ex-  
136 plicitly considering any optimization criterion) [27]. Most of these works  
137 address the best distribution of dimensional tolerances for optimizing some  
138 interpretation of the total production cost, while only a few contributions  
139 are available that account for geometrical tolerances as well.

## 140 2.2. Process and production planning stage

141 With a focus on assembled products, *assembly planning* (AP) creates  
142 a detailed assembly plan to craft a complete product from individual parts,  
143 considering aspects like product and part geometries, available resources (ma-  
144 chines, tools, fixtures, feeders, etc.) as well as technological constraints [28].  
145 When AP is solved by some automated techniques, following the manufactur-  
146 ing nomenclature, it is also called *computer-aided (assembly) process planning*  
147 (CAPP). Solving AP/CAPP requires making diverse types of decisions, and  
148 accordingly, it is usually solved by some decomposition approach. A typical  
149 decomposition scheme subdivides AP into the following three sub-problems  
150 [28]: (1) *Assembly Sequence Planning* (ASP), in which a sequence of (ex-  
151 pectantly technologically and geometrically feasible) assembly operations is  
152 computed; (2) *Assembly Line Balancing* (ALB), in which the assembly op-  
153 erations are assigned to assembly stations in such a way that station work-  
154 loads are balanced; and (3) *Assembly Path Planning* (APP), which computes  
155 collision-free paths for joining different parts or sub-assemblies in individual  
156 assembly operations.

157 Various data models have been proposed to enable the automatic gen-  
158 eration of process plans. Such models capture information on the target  
159 product, the applied equipment, as well as the manufacturing and assem-  
160 bly process. Specifically, data models have been proposed for describing the  
161 product structure and its features [29], product structure extended with tol-  
162 erances and quality [30], workers' abilities and ergonomics [31], fixtures and  
163 grasping [32]. In addition, models dedicated to specific fields have been pro-  
164 posed, such as the final assembly of automotive vehicles [33] and aircrafts  
165 [34]. Finally, there are ambitious initiatives, such as the ontology model  
166 by NIST [35], aiming at the generalization of the data models to assembled  
167 products, but these have not been put to practical use so far. However, pro-  
168 cess planning methods are still specialized for a given product family or an

169 assembly technology, and lack a feedback mechanism for product design, in  
170 particular for multi-product assembly.

### 171 *2.3. Computational methodology*

172 The combined product tolerance design, process planning, and production  
173 planning problem addressed in this paper is a complex combinatorial opti-  
174 mization problem. For production planning models similar in their structure,  
175 mathematical programming, and especially *mixed-integer linear programming*  
176 (MILP) approaches have been predominant and have proven efficient [36].  
177 Nevertheless, when solving complex and large instances of the problem, math-  
178 ematical programming approaches might be insufficient on their own, and the  
179 application of meta-heuristics may become the most effective approach.

180 A research direction of increasing importance in operations research is  
181 combining the strengths of mathematical programming and meta-heuristic  
182 approaches in so-called *matheuristics* [37]. The *large neighborhood search*  
183 (LNS) algorithm [38, 39] was motivated precisely by the need for combining  
184 exact solution methods with local search in applications where exact solu-  
185 tion approaches (e.g., branch-and-bound for a MILP or a constraint program)  
186 outperform pure meta-heuristic approaches, but still, they do not scale up to  
187 realistic problem sizes. LNS consists in constructing first an initial solution  
188 using some heuristic, and then, iteratively looking for improvements in some  
189 neighborhood of the current solution. However, the efficient exact solution  
190 approach (MILP in our case) makes it possible to search a very large, po-  
191 tentially exponential size neighborhood in each iterative step. LNS has been  
192 successfully applied to various fields of combinatorial optimization, including  
193 scheduling [40] and vehicle routing problems [41].

### 194 *2.4. Positioning of the paper*

195 A simplified version of the current problem was investigated by the au-  
196 thors in the recent paper [10]. A decomposition approach was introduced  
197 that separated the solution of the tolerance allocation and the assembly re-  
198 source configuration sub-problems. Case studies based on industrial data  
199 confirmed that the approach can effectively reduce production costs and im-  
200 prove investment efficiency.

201 The present paper addresses the generalization of the previous contribu-  
202 tion in industrially relevant directions, including a generic tolerance model  
203 with interrelated dimension chains, multiple target assembly processes, as

204 well as differentiating human and automated processes for achieving the re-  
205 quired precision by adjustments. The extension of the model also required  
206 the development of novel, efficient solution approaches instead of the decom-  
207 position scheme described in [10].

### 208 3. Problem statement

209 In the paper, a complex optimization problem is investigated with the aim  
210 of reducing the overall production-related costs through the proper combina-  
211 tion of product design, process planning and production planning decisions  
212 in high-precision multi-product assembly. In order to provide a comprehen-  
213 sible definition, the presentation of the overall problem is separated into four  
214 sub-sections as follows:

- 215 1. the *tolerance design* sub-problem, which involves the selection of the  
216 appropriate structural design alternatives and the assignment of toler-  
217 ance values to individual dimensions to meet the tolerance requirements  
218 on the assembled products;
- 219 2. the *assembly resource configuration* sub-problem, which aims to match  
220 forecast demand to assembly resources, considering existing and poten-  
221 tial future resource capabilities and capacities, as well as the process  
222 requirements according to the above defined product design;
- 223 3. the definition of the *production costs* and the *depreciation model* to  
224 characterize the quality of the solutions; and
- 225 4. a recapitulation of the *assumptions* made.

#### 226 3.1. Tolerance design sub-problem

227 The tolerance design sub-problem is responsible for selecting the appro-  
228 priate structural design alternative for each product from a list of alternatives  
229 given in the input, and for defining the tolerance values on the individual di-  
230 mensions in such a way that the tolerance requirements on the assembled  
231 product are satisfied, and the total production costs are minimized. While  
232 the satisfaction of tolerance requirements can be verified solely on the solu-  
233 tion of the tolerance design sub-problem, production costs also depend on  
234 the solution of the assembly resource configuration sub-problem.

235 The formal definition of the tolerance design sub-problem is the following.  
236 There is a set of products  $P$  to be produced in a common multi-product

Table 1: Notation (tolerance design).

<i>Indices, sets</i>	
$\delta$	Dimension (index)
$\Delta$	Dimension chain (index)
$d$	Design alternative (index)
$M(d)$	Set of manufactured dimensions of alternative $d$
$p$	Product (index)
$P$	Set of products
$D_{d,k}^+$	Set of dimension chains in alternative $d$ with adjustment by process $k$
$D_d^-$	Set of dimension chains in alternative $d$ without adjustment
<i>Input parameters</i>	
$\varphi_\Delta$	Design specification for dimension chain $\Delta$ [ $\pm$ mm]
$\varrho_\Delta$	Adjustment range of dimension chain $\Delta$ [ $\pm$ mm]
<i>Decision variables</i>	
$y_d$	Variable indicating that design alternative $d$ is selected for production
$\tau_\delta$	Tolerance on dimension $\delta$ [ $\pm$ mm]
$\bar{\tau}_{d,k}$	Adjustment precision on process $k$ required for assembling alternative $d$ [ $\pm$ mm]

237 assembly system, containing both *existing* and *new* products. The design  
238 of the existing products is fixed. In contrast, multiple candidate structural  
239 design alternatives, provided as input by a designer, are available for the new  
240 products, whose production begins during the planning horizon. For each  
241 new product, a single design alternative must be selected for production,  
242 and the design cannot be altered later.

243 Each given structural design alternative specifies the product structure  
244 in terms of nominal geometries of the parts and their relations, which defines  
245 the dimension chains  $\Delta$  of each alternative  $d$ . Different dimension chains can  
246 share common dimensions (i.e.,  $\Delta_1 \cup \Delta_2 \neq \emptyset$ ), which can be both adjusted  
247 and non-adjusted dimensions. Design requirements are given in terms of  
248 tolerance specifications  $\varphi_\Delta$  on each dimension chain  $\Delta$ .

249 At the same time, tolerance values  $\tau_\delta$  on individual dimensions  $\delta$  are not  
250 part of the input (this is why the design alternatives in the input are called  
251 only *structural* alternatives); instead, they must be calculated in the tolerance  
252 design sub-problem in such a way that the requirement specifications are  
253 satisfied for the selected design alternative  $d$  for each new product  $p$ . Without  
254 loss of generality, this paper assumes symmetric tolerances, where the upper  
255 (+) and lower (-) values are equal, e.g.  $\tau_\delta = \pm 0.01$  mm.

256 For design alternatives with adjustment, it is assumed that there is at  
257 most one adjusted dimension in each dimension chain. Moreover, adjustment

258 takes place after the assembly of all parts related to the involved dimensions,  
 259 and therefore, adjustment can compensate the deviation of all dimensions in  
 260 the chain (note that this assumption can be lifted with a minor generalization  
 261 of the model presented here). Consequently, tolerance specifications can be  
 262 met in two different ways:

- For dimension chains without adjustment, i.e., with fully defined connections between the parts, the tolerance values on the individual parts' dimensions need to be specified so as to guarantee that the stacked tolerance values satisfy the design specifications:

$$\sum_{\delta \in \Delta} \tau_{\delta} \leq \varphi_{\Delta}$$

- For dimension chains with adjustment option, the stacked tolerance value can be greater than the design specification. However, this is compensated by adjustment within a predefined range,  $\varrho_{\Delta}$ , which decreases the stacked tolerance. Nonetheless, the precision of the adjustment itself,  $r_{\Delta}$ , must be taken into account:

$$\sum_{\delta \in \Delta} \tau_{\delta} - \varrho_{\Delta} + r_{\Delta} \leq \varphi_{\Delta}$$

Moreover, the adjustment precision  $r_{\Delta}$  itself must satisfy the design specification:

$$r_{\Delta} \leq \varphi_{\Delta}$$

263 Finally, each adjusted dimension is unambiguously assigned to a certain  
 264 assembly process  $k$  that performs the adjustment. Accordingly, the tolerance  
 265 design sub-problem also involves the specification of the required adjustment  
 266 precision value  $\bar{r}_{d,k}$  for process  $k$ . The notation applied for the tolerance  
 267 design sub-problem is summarized in Table 1.

### 268 3.2. Assembly resource configuration sub-problem

269 The assembly resource configuration sub-problem is responsible for match-  
 270 ing the demand for the products to existing, new, or upgraded assembly  
 271 resources. This involves the following types of decisions:

- 272 • Deciding on the potential construction of new assembly lines.

- 273 • Automating selected processes on assembly lines.
- 274 • Upgrading selected automated processes to improve their adjustment  
275 precision (while the precision of human processes is assumed to be  
276 fixed).
- 277 • Assigning products (with their corresponding design alternatives) to  
278 assembly lines in such a way that all capacity and capability require-  
279 ments are satisfied.

280 Formally, each product  $p \in P$  in each period  $t \in T$  must be assigned to  
281 some assembly lines. Products can be assigned to at most  $\Pi$  assembly lines  
282 at a time (noting that  $\Pi = 1$  in most of the use cases investigated), which can  
283 be either existing lines or newly built lines. On the other hand, an arbitrary  
284 number of products can share the same assembly line.

285 Modifying the product-line assignment over the horizon is allowed, how-  
286 ever, this comes with a changeover cost of  $c^X$  and a changeover time of  $a^X$   
287 on the newly assigned line. Hence, the capacity constraint on assembly line  
288  $l$  requires that the total assembly time of the products,  $g_{p(d)t} a_d \xi_{dlt}$  (where  
289  $g_{p(d)t}$  is the forecasted demand,  $a_d$  is the per unit assembly time, and decision  
290 variable  $\xi_{dlt}$  denotes the fraction of the demand assigned to the given line),  
291 plus the potential changeover times,  $a^X u_{dlt}$  (where auxiliary variable  $u_{dlt}$  in-  
292 dicates if there is a changeover to design alternative  $d$  on the line), cannot  
293 exceed the fixed capacity  $q_l$  of the lines:

$$\sum_d (g_{p(d)t} a_d \xi_{dlt} + a^X u_{dlt}) \leq q_l$$

294 Assembly lines consist of multiple stations that execute different assembly  
295 processes, among which focus is given to high-precision adjustment processes  
296 necessary for setting the adjusted dimensions of the selected design alterna-  
297 tives. Each process  $k$  can be performed by a human operator, or alternatively,  
298 it can be automated for a given automation cost. Nevertheless, once a pro-  
299 cess is automated, it cannot be downgraded to a human process later. A  
300 combination of human and automated processes is also allowed on the same  
301 assembly line.

302 Each process is further characterized by its achievable adjustment preci-  
303 sion  $b_{ltk}$ . The adjustment precision of human processes is a fixed value of  
304  $b^H$ . On the contrary, the initial adjustment precision  $b_{l0k}$  of an automated  
305 process  $k$ , which may be insufficient to assemble the design alternatives with

Table 2: Notation (assembly resource configuration sub-problem).

<i>Indices, sets</i>	
$t$	Time period (index)
$l$	Assembly line (index)
$r$	Adjustment precision cost function breakpoint index
$L$	Set of assembly lines
$T$	Set of time periods
$L^{\text{new}}$	Set of potential new lines
<i>Input parameters</i>	
$g_{pt}$	Order amount for product $p$ in period $t$ [pcs.]
$a_d$	Processing time of design alternative $d$ on assembly lines [s/pcs.]
$a^X$	Changeover time on assembly lines [s/pcs.]
$q_l$	Nominal capacity of (existing or potential new) line $l$ [s]
$b_{l0k}$	Initial adjustment precision of process $k$ of line $l$ [ $\pm$ mm]
$\underline{b}$	Possible best adjustment precision of assembly lines [ $\pm$ mm]
$b^H$	Adjustment precision ability of the human operators [ $\pm$ mm]
$\Pi$	Max. number of parallel lines for processing the same product [pcs.]
$T^D$	Useful life of assembly lines in the depreciation model [time periods]
$C_{lk}^{P0}$	Adjustment precision cost of line $l$ in its initial state [\$]
<i>Decision variables</i>	
$x_{dlt}$	Variable indicating that design alternative $d$ is assigned to line $l$ in period $t$
$\xi_{dlt}$	Fraction of the demand for design alternative $d$ assigned to line $l$ in period $t$
$u_{dlt}$	Variable indicating that design alternative $d$ is reassigned to line $l$ in period $t$
$z_{lt}$	Variable indicating that new line $l$ is installed in period $t$
$b_{ltk}$	Adjustment precision of process $k$ of line $l$ in period $t$ [ $\pm$ mm]
$v_{ltk}$	Variable indicating that process $k$ of line $l$ is automated in period $t$

306 adjustment, can be upgraded to  $b_{ltk}$  with  $b_{l0k} \geq b_{ltk} \geq \underline{b}$  by the enhance-  
 307 ment of the automated equipment. It should be noted that  $b^H < b_{ltk}$  is also  
 308 allowed, which implies that automation with a substandard equipment may  
 309 deteriorate the precision of the assembly process.

310 Then, a selected design alternative can be assigned to a line  $l$  if the adjust-  
 311 ment precision of the line is at least as good as the precision required by that  
 312 design alternative for every process  $k$ , i.e.,  $\bar{r}_{d,k} \geq b_{ltk} \quad \forall k, t$ . The notation  
 313 applied for the assembly resource configuration sub-problem is summarized  
 314 in Table 2.

### 315 3.3. Production costs and depreciation model

316 The objective is minimizing the *total production costs*, which comprises  
 317 costs related to parts manufacturing, assembly, and investments. *Manufac-*  
 318 *turing costs* are composed of the fixed, per unit base manufacturing cost  $c_d^{T0}$

319 of the selected design alternative  $d$  and a tolerance cost, calculated as the  
 320 sum of the costs of manufacturing the individual dimensions with the spec-  
 321 ified tolerances. Hence, the per unit manufacturing cost  $C_p^M$  of product  $p$ ,  
 322 with selected design alternative  $d$  and manufactured dimensions  $M(d)$ , can  
 323 be calculated as:

$$C_p^M = c_d^{T0} + \sum_{\delta \in M(d)} C_\delta^T$$

324 The tolerance costs of the individual dimensions are approximated by  
 325 convex piecewise linear functions for each dimension  $\delta$ , specified with the  
 326 breakpoints of the functions. The  $x$  coordinates  $C_r^{T[x]}$  of the function are  
 327 the tolerance values  $\tau_\delta$ , while the  $y$  coordinates  $C_r^{T[y]}$  provide the costs of  
 328 manufacturing dimension  $\delta$  to a given tolerance  $\tau_\delta$ . Accordingly, the tolerance  
 329 cost  $C_\delta^T(\tau_\delta)$  can be calculated using the following formula:

$$C_\delta^T(\tau_\delta) = \max_{r \geq 2} \left( C_{r-1}^{T[y]} \frac{C_r^{T[x]} - \tau_\delta}{C_r^{T[x]} - C_{r-1}^{T[x]}} + C_r^{T[y]} \frac{\tau_\delta - C_{r-1}^{T[x]}}{C_r^{T[x]} - C_{r-1}^{T[x]}} \right)$$

330 *Assembly costs* are composed of the operation costs  $c_l^0$  of the assembly  
 331 line per units produced. In addition to that, a labor cost of  $c^H$  per unit is  
 332 charged for the manual processes on the lines. Finally, each changeover on  
 333 the lines is penalized with a changeover cost of  $c^X$ .

334 Further costs are related to *investments* into new or upgraded assembly  
 335 equipment. New lines can be built for a base investment cost of  $c^L$ , which  
 336 includes the installation of a manual assembly line. Processes on the exist-  
 337 ing or newly built lines can be automated for a given automation cost of  $c_k^A$   
 338 for each process  $k$ . Further, the precision of the automated processes can  
 339 be upgraded, which is captured by a convex piecewise linear function, again  
 340 given its breakpoints. Similarly to the tolerance cost function, values  $C_{rk}^{P[x]}$   
 341 on the  $x$  axis provide the precision, while values  $C_{rk}^{P[y]}$  on the  $y$  axis define the  
 342 corresponding costs for process  $k$ . The investment costs related to a certain  
 343 precision upgrade of process  $k$  can be calculated as the difference of equip-  
 344 ment values realized in two subsequent periods, i.e.,  $C_{ltk}^P(b_{ltk}) - C_{l0k}^P(b_{l0k})$ .

345 All investment costs—including the installation of new lines, upgrading  
 346 the adjustment precision or the level of automation—are calculated by using  
 347 a *linear depreciation model* with a useful life of  $T^D$ . The notation for cost  
 348 components is summarized in Table 3.

349 Finally, the *total production cost* is calculated as the sum of the parts  
350 manufacturing cost, the assembly line operation cost, the assembly labor  
351 cost, the changeover cost, as well as the investment costs related to new line  
352 installation, upgrades in adjustment precision, and in the level of automation.  
353 When solving the problem, the solution that minimizes this cost is sought.

354 Minimizing the above complex cost function captures the problem of find-  
355 ing the best tradeoff between different approaches to reaching the desired  
356 product qualities. Strict precision requirements can be satisfied by manu-  
357 facturing precision parts (which leads to high manufacturing costs) or by  
358 incorporating an appropriate adjustment mechanism in the product design  
359 (which comes with lower manufacturing but higher assembly costs). Like-  
360 wise, the selection of human and automated assembly resources that can  
361 serve the precision requirements is a challenging problem. Moreover, the  
362 synergies between different products sharing the same assembly equipment  
363 must be exploited. Finally, it is emphasized that all cost components are  
364 expressed in monetary terms, and therefore can be summarized to constitute  
365 a single objective function, and hence, there is no need for considering com-  
366 plex multi-criteria optimization. Nevertheless, it must be ensured that the  
367 time horizon is long enough and demand forecasts are sufficiently reliable to  
368 capture a realistic demand volume for all products.

### 369 3.4. Assumptions

370 This section recapitulates the assumptions made in the above model, both  
371 during tolerance design and assembly resource configuration:

- 372 • The model focuses on the assembly of precision products, where the  
373 costs related to achieving the desired tolerances are crucial both in  
374 parts manufacturing and in assembly.
- 375 • Design requirements are expressed in terms of dimensional tolerance  
376 specifications on each dimension chain.
- 377 • There is at most one adjusted dimension in each dimension chain.
- 378 • Adjustment happens after the assembly of all related parts, and hence,  
379 it compensates the deviation of all dimensions in the chain (though,  
380 this assumption can be lifted with a minor extension of the model).
- 381 • In parts manufacturing, tolerance costs are captured by convex, piece-  
382 wise linear functions assigned to individual dimensions, see, e.g., [42].

Table 3: Notation (costs).

<i>Cost parameters</i>	
$c_d^{T0}$	Base manufacturing cost of design alternative $d$ [\$]
$c_l^0$	Cost of operating line $l$ for a unit time [\$/s]
$c^X$	Cost of a changeover on assembly lines unit time [\$/pcs.]
$c^L$	Cost of installing a new line [\$/line]
$c_k^A$	Fix cost of automating process $k$ [\$/process]
$c^H$	Unit cost of human labor [\$/min.]
$(C_r^{T[x]}, C_r^{T[y]})$	Breakpoint $r \in R$ of the tolerance cost function [(± mm,\$)]
$(C_{rk}^{P[x]}, C_{rk}^{P[y]})$	Breakpoint $r \in R$ of the adjustment precision cost function of process $k$ [(± mm,\$)]
<i>Cost function components</i>	
$C_{ltk}^P$	Precision capability value of process $k$ of line $l$ in period $t$ [\$]
$C_\delta^T$	Cost of manufacturing dimension $\delta$ to the selected tolerance [\$]
$C^M$	Unit cost of manufacturing product $p$ [\$/pcs.]
$C^M$	Parts manufacturing cost [\$]
$C^L$	Assembly lines operation cost [\$]
$C^I$	Installation cost of new assembly lines [\$]
$C^P$	Investment cost of upgrading the adj. prec. of assembly lines [\$]
$C^X$	Changeover cost on assembly lines [\$]
$C^H$	Human labor cost [\$]
$C^A$	Automation cost [\$]

- 383     • Likewise, the investment cost of automated machinery for a given as-  
384     sembly process can be described by a convex, piecewise linear function  
385     of the desired precision.
- 386     • Full interchangeability of parts is assumed, i.e., there are no defective  
387     items and no selective assembly is required.
- 388     • A sufficiently precise demand forecast is available for the products.

#### 389 4. Solution approach

390     Two alternative but related solution approaches have been investigated  
391     and implemented to address the above defined problem:

- 392     • A monolithic *mixed-integer linear programming* (MILP) formulation,  
393     which is a declarative representation of the problem at hand, and which  
394     can be solved directly using commercial MILP solvers. These solvers  
395     use branch-and-bound search for solving the MILP formulation, which

396 implies that the approach is *exact*, i.e., it constructs proven, *exact opti-*  
 397 *mal* solutions if sufficient computational time is available. On the other  
 398 hand, this approach can be unsuitable for very large problem instances.

- 399 • A *large neighborhood search* (LNS) algorithm based on the same MILP  
 400 formulation, which combines the above branch-and-bound solution ap-  
 401 proach with local search. This combination is expected to scale up bet-  
 402 ter to very large problem instances, however, like typical local search  
 403 approaches, it cannot provide any guarantee on the quality of the so-  
 404 lution found.

#### 405 4.1. Monolithic MILP formulation

406 The above defined problem can be encoded in the form of a MILP as  
 407 presented below. In addition to classical linear constraints, this formulation  
 408 makes use of so-called *indicator constraints*, a modelling utility offered by  
 409 various commercial MILP solvers including FICO Xpress or IBM CPLEX  
 410 for expressing logical combinations of constraints. An indicator constraint  
 411 of the form  $x \Rightarrow c$ , where  $x$  is a binary variable and  $c$  is a linear constraint,  
 412 states that if  $x$  takes a value of 1, then constraint  $c$  must hold. From the  
 413 conventional mathematical programming toolkit, one could use so-called *big-*  
 414 *M constraints* to express the same logical relations, however, indicator con-  
 415 straints result in a more readable model, more robust behavior, and improved  
 416 computational efficiency by allowing the MILP solver to calculate tight coef-  
 417 ficients for variable  $x$  in the constraint, even during the solution process [43].  
 418 Hence, the overall problem formulation is as follows.

Minimize

$$C^M + C^H + C^L + C^X + C^I + C^P + C^A \quad (1)$$

subject to

$$\sum_{\delta \in \Delta} \tau_{\delta} \leq \varphi_{\Delta} \quad \forall \Delta \in D_d^- \quad (2)$$

$$\sum_{\delta \in \Delta} \tau_{\delta} - \varrho_{\Delta} + r_{\Delta} \leq \varphi_{\Delta} \quad \forall d, k, \Delta \in D_{d,k}^+ \quad (3)$$

$$r_{\Delta} \leq \varphi_{\Delta} \quad \forall d, k, \Delta \in \bigcup_k D_{d,k}^+ \quad (4)$$

$$C_1^{T[x]} \leq \tau_\delta \leq C_R^{T[x]} \quad \forall \delta \quad (5)$$

$$\bar{r}_{d,k} \leq r_\Delta \quad \forall d, k, \Delta \in D_{d,k}^+ \quad (6)$$

$$C_\delta^T \geq \left( C_{r-1}^{T[y]} \frac{C_r^{T[x]} - \tau_\delta}{C_r^{T[x]} - C_{r-1}^{T[x]}} + C_r^{T[y]} \frac{\tau_\delta - C_{r-1}^{T[x]}}{C_r^{T[x]} - C_{r-1}^{T[x]}} \right) \quad \forall \delta, r \geq 2 \quad (7)$$

$$\sum_{d \mid p(d)=p} y_d = 1 \quad \forall p \quad (8)$$

$$\sum_l \xi_{dlt} = y_d \quad \forall d, t : g_{p(d)t} > 0 \quad (9)$$

$$\sum_l x_{dlt} \leq \Pi y_d \quad \forall d, t : g_{p(d)t} > 0 \quad (10)$$

$$\xi_{dlt} \leq x_{dlt} \quad \forall d, l, t : g_{p(d)t} > 0 \quad (11)$$

$$x_{dlt} \leq z_{lt} \quad \forall d, l \in L^{\text{new}}, t \quad (12)$$

$$(1 - v_{ltk}) \Rightarrow (\bar{r}_{d,k} \geq b^H x_{dlt}) \quad \forall d, l, t, k \quad (13)$$

$$v_{ltk} \Rightarrow (b_{l0}(1 - x_{dlt}) + \bar{r}_{d,k} \geq b_{ltk}) \quad \forall d, l, t, k \quad (14)$$

$$v_{ltk} \geq v_{l(t-1)k} \quad \forall l, t, k \quad (15)$$

$$u_{dlt} \geq x_{dlt} - x_{d(t-1)l} \quad \forall d, l, t \quad (16)$$

$$\sum_d (g_{p(d)t} a_d \xi_{dlt} + a^X u_{dlt}) \leq q_t \quad \forall l, t \quad (17)$$

$$z_{lt} \leq z_{l(t-1)} \quad \forall l \in L^{\text{new}}, t \quad (18)$$

$$\underline{b} \leq b_{ltk} \leq b_{l(t-1)} \quad \forall l, t \quad (19)$$

$$C_{ltk}^P \geq \left( C_{r-1k}^{P[y]} \frac{C_{rk}^{P[x]} - b_{ltk}}{C_{rk}^{P[x]} - C_{r-1k}^{P[x]}} + C_{rk}^{P[y]} \frac{b_{ltk} - C_{r-1k}^{P[x]}}{C_{rk}^{P[x]} - C_{r-1k}^{P[x]}} \right) \quad \forall l, k, r \geq 2 \quad (20)$$

$$C_{ltk}^P \geq C_{l(t-1)k}^P \quad \forall l, t, k \quad (21)$$

$$C^P = \sum_{ltk} \frac{1}{TD} (C_{ltk}^P - C_{ltk}^{P0}) \quad (22)$$

$$C_{ltk}^H \geq c^H \left( \sum_d (g_{p(d)t} a_d \xi_{dlt} + a^X u_{dlt}) - (1 - v_{ltk}) q_l \right) \quad \forall l, t \quad (23)$$

$$C^H = \sum_{ltk} C_{ltk}^H \quad (24)$$

$$y_d \Rightarrow \left( C_p^M \geq c_d^{T0} + \sum_{\delta \in M(d)} C_\delta^T \right) \quad \forall d, p = p(d) \quad (25)$$

$$C^M = \sum_{pt} (C_p^M g_{pt}) \quad (26)$$

$$C^L = \sum_{dlt} (c_l^0 g_{p(d)t} a_d \xi_{dlt}) \quad (27)$$

$$C^X = c^X \sum_{dlt} u_{dlt} \quad (28)$$

$$C^I = \frac{c^L}{T^D} \sum_{lt} z_{lt} \quad (29)$$

$$C^A = \sum_{ltk} (v_{ltk} - v_{l(t-1)k}) \frac{c_k^A \min(T^D, T - t + 1)}{T^D} \quad (30)$$

$$x_{dlt}, y_d, u_{dlt}, z_{lt}, v_{ltk} \in \{0, 1\} \quad \forall d, l, t \quad (31)$$

$$\xi_{dlt}, b_{ltk} \geq 0 \quad \forall d, l, t \quad (32)$$

419

420 The objective (1) stands for minimizing the total cost, composed of the  
 421 parts manufacturing cost  $C^M$ , the assembly labor cost  $C^H$ , the assembly line  
 422 operation cost  $C^L$ , the changeover cost  $C^X$ , the new line installation cost  $C^I$ ,  
 423 the lines' precision upgrade costs  $C^P$ , and the assembly line automation cost  
 424  $C^A$ .

425 The tolerance assignment sub-problem is addressed in constraints (2)-(7),  
 426 whose solution is relevant only for the design alternatives selected for produc-  
 427 tion. Constraint (2) requires that the stacked tolerance along any dimension  
 428 chain without adjustment amounts to at most the design specification for  
 429 the given chain. In contrast, for chains with adjustment, the adjustment  
 430 mechanism can compensate an error equal to the adjustment range of the  
 431 mechanism minus its adjustment precision (3). At the same time, the ad-  
 432 justment precision itself cannot be looser than the tolerance specification of  
 433 the chain (4). Bounds for the individual tolerances must be in line with

434 technological limits (5). The adjustment precision requirement of a design  
 435 for any process  $k$  is at least as strict as the precision of the adjustments per-  
 436 formed during that process (6). Finally, the cost related to the tolerance on  
 437 an individual dimension is determined by the piecewise linear cost function  
 438  $C_{\delta}^T(\tau_{\delta})$  (7).

439 The second part of the MILP focuses on the selection of the design al-  
 440 ternatives, the assignment of the design alternatives to assembly lines, and  
 441 the configuration of these lines. Equality (8) states that for each product,  
 442 exactly one design alternative must be selected for production. Constraints  
 443 (9) and (10) ensure that the complete demand for the selected design alter-  
 444 natives is distributed among at most  $\Pi$  assembly lines in each time period  
 445 where there is nonzero demand for the given product. Moreover, a fraction  
 446 of the demand for design alternative  $d$  can be assigned to a line  $l$  only if  $d$   
 447 is assigned to  $l$  in the given period (11). Products can be assigned to new  
 448 lines only if the lines are already installed (12). Furthermore, the adjustment  
 449 precision of the line must be at least as good as the precision required by  
 450 the design alternative, both in case of human (13) and automated processes  
 451 (14). Automated processes cannot be downgraded to manual (15).

452 Constraint (16) relates the changeover variables to the assignment vari-  
 453 ables. The capacity constraint (17) states that the sum of processing times  
 454 and changeover times on an assembly line, either existing or newly built,  
 455 cannot exceed the line capacity. Investments related to new line installation  
 456 (18) and adjustment precision upgrade (19) are performed in a given period  
 457 of time, and they cannot be undone later.

458 Inequality (20) calculates the adjustment precision costs of the individ-  
 459 ual lines in each time period. These adjustment precision costs increase  
 460 monotonously over time (21). From these values, the total adjustment pre-  
 461 cision upgrade cost is computed by equality (22), by subtracting the cost  
 462 of the initial lines from the extended lines, also accounting for depreciation.  
 463 Similarly, inequality (23) calculates the per period per line labor cost, and  
 464 equation (24) sums these values to compute the total labor cost.

465 Constraint (25) calculates the unit manufacturing cost of a product as a  
 466 sum of the base manufacturing cost and the total tolerance cost on the man-  
 467 ufactured dimensions for the selected design alternative. Then, equations  
 468 (26)-(30) calculate the manufacturing, the line operation, the changeover, the  
 469 new line installation, as well as the line automation costs, respectively. Fi-  
 470 nally, constraints (31) and (32) define the variables as binary or non-negative  
 471 continuous.

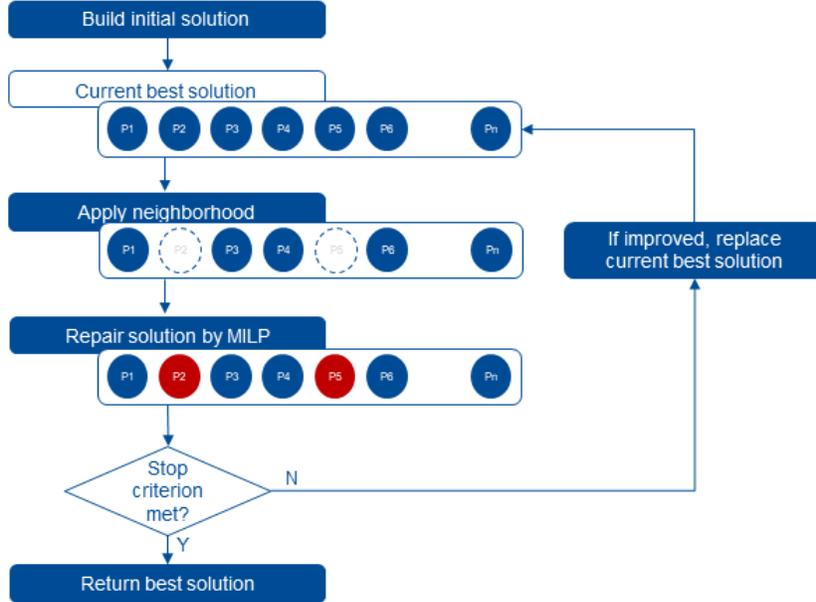


Figure 2: Flowchart of the LNS approach.

472 *4.2. Large neighborhood search algorithm*

473 In order to solve very large instances of the above problem, an LNS  
 474 matheuristic solution approach was implemented, which combines mathe-  
 475 matical programming for solving the above MILP representation with local  
 476 search techniques. The application of LNS to the particular problem required  
 477 adapting the approach in both of its two main steps: the construction of the  
 478 initial solution, and the iterative exploration of the local neighborhood.

479 For constructing an initial solution, a so-called *Russian Doll* approach  
 480 has been applied: a hierarchy of embedded time intervals is defined as  $T_1 \subset$   
 481  $T_2 \subset \dots \subset T_K = T$  with  $T_k = [1, k\Delta T]$ . In step  $k$  of the algorithm, the  
 482 optimal solution for time interval  $T_k$  is computed subject to the constraint  
 483 that the head of the solution corresponding to interval  $T_{k-1}$  matches the  
 484 earlier solution for  $T_{k-1}$ . During all experiments, the value of  $\Delta T = 5$  and a  
 485 time limit of 300 seconds was used.

486 In the iterative step of LNS, an improved solution is looked for by re-  
 487 solving the original problem with the added constraints that, for a subset of  
 488 the products ( $N - 2$  products in the current implementation), the selection  
 489 of the design alternative and the assignment to assembly lines cannot be

490 modified. For the remaining products (2 products in the implementation),  
491 both the selection of the design alternative and the assignment to assembly  
492 lines is reconsidered by solving the restricted version of the original MILP  
493 model to optimality (or stopping the MILP solver at a given time limit,  
494 300 seconds in the experiments). The algorithm replaces the previous best  
495 solution by the current iterative solution if and only if the current solution  
496 is an improvement over the previous best solution. LNS terminates when  
497 all neighborhoods have been searched or it reaches a pre-defined time limit,  
498 3600 seconds in the experiments reported. The flowchart of the algorithm is  
499 presented in Figure 2.

500 It is noted that various alternative algorithms have been implemented and  
501 evaluated both for constructing the initial solution and for the iterative step,  
502 but a decision has been made for the above procedures due to their simplicity  
503 and efficiency, as it will be shown below in the experimental evaluation.

## 504 5. Experimental evaluation

505 In this case study, the viability of the proposed method in multi-product  
506 high-precision assembly is investigated from the viewpoints of computational  
507 efficiency and industrial effectiveness. The following subsections first intro-  
508 duce the sample product and the production environment. Then, the com-  
509 putational efficiency of the two solution approaches, monolithic MILP and  
510 LNS, is investigated and compared. Finally, a real industrial case study is  
511 presented in detail.

### 512 5.1. Production environment

513 The experimental evaluation shown below in Sections 5.2 and 5.3 are  
514 based on sample data originating from the industry, and involves a product  
515 family that contains eight different products, three of which are new. While  
516 the designs of the five existing products are given and cannot be modified,  
517 the designs for the three new products can be selected from six structurally  
518 different design alternatives, and their tolerance allocation should also be op-  
519 timized. Each target product in the family consists of several parts and has  
520 two design specifications, ( $\varphi_{f2}$  and  $\varphi_{f3}$ ), which are guided by dimensional  
521 tolerances between the parts. An overview of the product structure is shown  
522 in Figure 3. Structural design alternatives differ in several ways: the parts  
523 called *house* and *cover* can be integrated ( $d_{INT}$ ) or separated ( $d_{SEP}$ ); and it  
524 is possible to incorporate or omit the adjustment mechanisms between the

525 *house* and the *cover* (only for  $d_{\text{SEP}}$ ), or between the *house* and the *stop-*  
 526 *per* (both  $d_{\text{INT}}$  and  $d_{\text{SEP}}$ ). The combinations of these choices define the six  
 527 structural design alternatives.

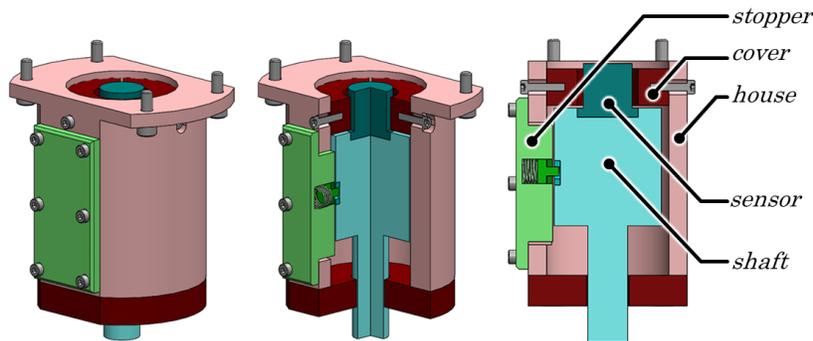


Figure 3: One structural design alternative,  $d_{\text{SEP}}$  with two adjustment mechanisms, for the sample product.

528 The process plan consists of three target processes, and it is presented in  
 529 Figure 4 for each of the design alternatives. *Process 1* assembles the *shaft* and  
 530 the *sensor*, without the option of adjustment. Hence, the resulting assembly  
 531 tolerance is defined by the stacked tolerance of the parts,  $\delta_{A1} = \delta_K + \delta_J$ .

532 *Process 2* involves the assembly of the *house* and the *shaft*, and it de-  
 533 termines one of the final design specifications,  $\varphi_{f2}$ . Without adjustment,  
 534 the design specification must be satisfied by the stacked tolerance on the  
 535 involved individual dimensions, i.e.,  $\varphi_{f2} \geq \delta_D + \delta_{A1} = \delta_D + \delta_K + \delta_J$  for  $d_{\text{INT}}$   
 536 and  $\varphi_{f2} \geq \delta_F + \delta_G + \delta_{A1} = \delta_F + \delta_G + \delta_K + \delta_J$  for  $d_{\text{SEP}}$ .

537 For both possible adjustment mechanisms in the sample product, the ad-  
 538 justment range of the mechanism is larger than the stacked tolerance on the  
 539 dimensions in the same dimension chain. Therefore, in case of adjustment in  
 540 *Process 2*, the design specification must be satisfied directly by the adjust-  
 541 ment precision of the machine or a skilled worker who performs the given  
 542 process, i.e.,  $\varphi_{f2} \geq r_{\Delta 2}$ .

543 Finally, *Process 3*, which assembles the *stopper* sub-assembly to the *house*,  
 544 is responsible for meeting the design specification  $\varphi_{f3}$ , with or without ad-  
 545 justment. Observe that the two dimension chains related to  $\varphi_{f2}$  and  $\varphi_{f3}$   
 546 share common dimensions, e.g.,  $A1$ .

547 For the assessment of production costs, 15 periods demand forecast data  
 548 is considered as an input. To satisfy the production volume, there are three

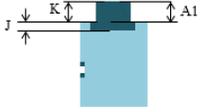
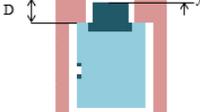
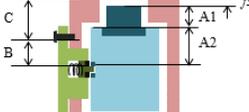
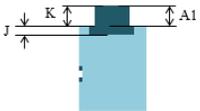
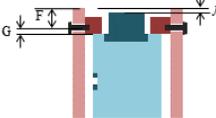
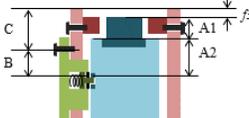
	Process 1 Shaft and sensor	Process 2 House and shaft	Process 3 House and stopper sub-assembly	
$d_{\text{INT}}$				
Adjustment	w/o	$\delta_{A1} = \delta_K + \delta_J$	$\varphi_{f2} \geq \delta_D + \delta_{A1}$	$\varphi_{f3} \geq \delta_C + \delta_B + \delta_{A1} + \delta_{A2}$
	w	---	---	$\varphi_{f3} \geq r_{\Delta 3}$
$d_{\text{SEP}}$				
Adjustment	w/o	$\delta_{A1} = \delta_K + \delta_J$	$\varphi_{f2} \geq \delta_F + \delta_G + \delta_{A1}$	$\varphi_{f3} \geq \delta_C + \delta_B + \delta_{A1} + \delta_{A2}$
	w	---	$\varphi_{f2} \geq r_{\Delta 2}$	$\varphi_{f3} \geq r_{\Delta 3}$

Figure 4: Assembly process of the sample product and the dimension chains related to each process. Design alternatives are defined by different combinations of the integrated ( $d_{\text{INT}}$ ) or the separated ( $d_{\text{SEP}}$ ) structure of the *house* and the *cover*, and designs with (w) or without (w/o) adjustment mechanisms related to each dimension chain.

549 existing assembly lines, with or without adjustment equipment and different  
550 adjustment precisions. As the production volume increases, two additional  
551 assembly lines have to be built by the end of the horizon.

## 552 5.2. Assessment of computational efficiency

553 In order to compare the computational efficiency of the two proposed  
554 mathematical models on a large set of instances with controllable sizes, ar-  
555 tificially generated problem instances were derived from the above original  
556 dataset by applying random perturbations. The problem size was controlled  
557 by two parameters: the length of the planning horizon  $|T| \in \{10, 20, 40\}$   
558 and the number of products  $|P| \in \{4, 8, 16\}$ , with half of the products being  
559 new, while the other half existing products. The number of structural design  
560 alternatives per products was fixed to 6, with 4 alternatives requiring adjust-  
561 ment. These settings resulted in  $3 \cdot 3 = 9$  combinations of the parameters,  
562 and for each combination, five different random instances were generated,

563 leading to 45 problem instances in total. Random perturbations were ap-  
 564 plied to the production volumes, maintaining realistic demand profiles, e.g.,  
 565 increasing production volumes for new products and decreasing volumes for  
 566 some old products (Figure 5). The realistic nature of the random instances  
 567 was maintained by keeping the structure, i.e., the dimension chains of the  
 568 design alternatives unchanged. They are results of design engineering work  
 569 that could be hardly captured by random instance generators.

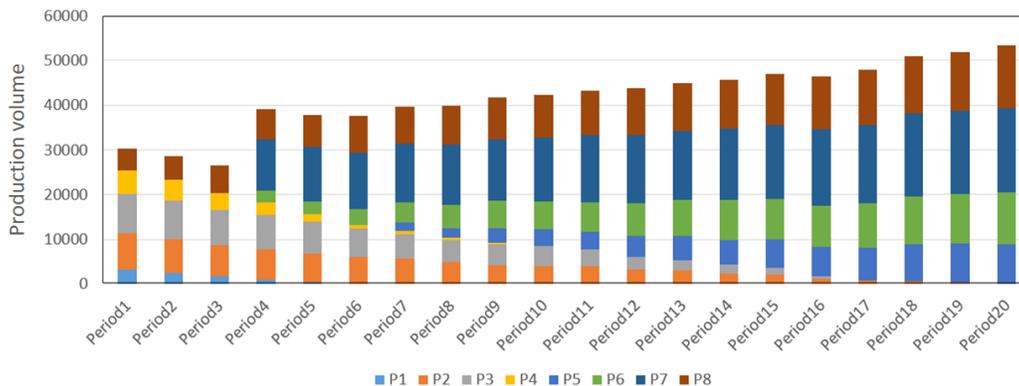


Figure 5: Demand volumes in a randomly generated instance with 8 products.

570 All the reported experiments were run with a time limit of one hour for  
 571 both the LNS and the monolithic MILP approaches. The experiments were  
 572 run on a virtual (cloud) computer with Linux operating system, using the  
 573 FICO Xpress 8.2 commercial MILP solver.

574 The results of the experiments are displayed in Table 4, where each row  
 575 contains combined results for the 5 instances for a given  $|P|$  and  $|T|$ . Sep-  
 576 arately for the MILP and the LNS approaches, the table shows the number  
 577 of instances out of 5 where a feasible solution was found (column *Sol*), the  
 578 number of instances solved to optimality (column *Opt*), the average and max-  
 579 imum optimality gaps (columns *Avg. gap* and *Max. gap*), and the average  
 580 computation time (*Avg. time*). For each instance and solution approach,  
 581 the optimality gap was calculated as  $(UB-LB)/LB$ , where UB is the upper  
 582 bound (solution value) found by the given approach, and LB is the lower  
 583 bound computed by MILP. Since MILP could not find any solution for some  
 584 of the largest instances (though, it could always compute a lower bound),  
 585 gaps are computed only for the instances with a feasible solution with the

586 given approach. Finally, it is noted that the meaning of optimality in the  
587 table is slightly different for MILP and LNS: while MILP is an exact solution  
588 approach that can actually prove the optimality of a solution (corresponding  
589 to a gap of 0%), LNS alone cannot yield such a proof; instead, it can be  
590 observed a posteriori that the LNS solution matches the value of the MILP  
591 lower bound.

592 The results show that the smallest instances ( $|P|=4$ ) were easily solvable  
593 by the MILP to proven optimality with a single exception. In contrast,  
594 for most of the medium-sized instances ( $|P|=8$ ), MILP terminated with a  
595 sub-optimal solution after one hour of computation, with average gaps of  
596 1.8%-6.6% and a maximum gap of 8.9%. The largest instances were indeed  
597 challenging for MILP: one third of the instances could not be solved at all,  
598 and even for the solvable instances, MILP terminated with considerable gaps  
599 (average gaps of 3.8%-37.9%, and a gap of 69.5% for one of the instances).

600 Two main observations can be made on the performance of the LNS. First,  
601 on the 19 instances with known optimum (all instances with  $|P|=4$ , and some  
602 with  $|P|=8$ ) one can observe that LNS results in close-to-optimal solutions  
603 with an average error of 0.0034%, which is an extremely good performance  
604 from a matheuristic approach. For the 26 more challenging instances with-  
605 out a known optimal solution, LNS clearly outperformed the exact MILP  
606 approach. It found reasonable solutions for 5 instances where MILP could  
607 not find a solution at all. Even when MILP could find a feasible solution,  
608 LNS improved that solution by 4.5% on average, and by 56.6% in an extreme  
609 case. There was a single instance where MILP could find a somewhat bet-  
610 ter solution than LNS, by 0.16%. Moreover, LNS typically required lower  
611 computation times than MILP both for the small and the large instances.

### 612 5.3. Assessment of industrial effectiveness

613 The experiments presented here were carried out on a real industrial prod-  
614 uct family, similar in its size, complexity, and the involved assembly processes  
615 to the product presented in detail above. This product family consists of six  
616 products, two of which are new. There are two assembly processes, each  
617 with an independent final tolerance specification. The specifications differ  
618 for each product, and the range of specifications starts from 0.15 mm, which  
619 is also the best adjustment precision that a human operator can achieve.  
620 These specifications must be reached using parts with tolerances on individ-  
621 ual dimensions starting from 0.05 mm. There are design alternatives with

Table 4: Comparison of the MILP and the LNS solution approaches. The best values over the different approaches are highlighted in bold.

P	T	MILP					LNS				
		Sol	Opt	Avg. gap	Max. gap	Avg. time	Sol	Opt	Avg. gap	Max. gap	Avg. time
4	10	<b>5</b>	<b>5</b>	0.0%	0.0%	<b>25</b>	<b>5</b>	<b>5</b>	0.0%	0.0%	30
	20	<b>5</b>	<b>4</b>	0.3%	1.6%	780	<b>5</b>	<b>4</b>	<b>0.3%</b>	<b>1.3%</b>	<b>65</b>
	40	<b>5</b>	<b>5</b>	0.0%	0.0%	551	<b>5</b>	<b>5</b>	0.0%	0.0%	<b>102</b>
8	10	<b>5</b>	0	6.6%	8.9%	3 600	<b>5</b>	0	<b>5.5%</b>	<b>6.9%</b>	<b>168</b>
	20	<b>5</b>	<b>2</b>	3.6%	8.3%	2 960	<b>5</b>	0	<b>3.3%</b>	<b>8.0%</b>	<b>385</b>
	40	<b>5</b>	<b>3</b>	1.8%	5.0%	1 950	<b>5</b>	2	<b>1.5%</b>	<b>4.2%</b>	<b>1 004</b>
16	10	<b>5</b>	0	3.8%	4.6%	3 600	<b>5</b>	0	<b>3.5%</b>	<b>4.2%</b>	<b>1 616</b>
	20	3	0	11.8%	22.7%	3 600	<b>5</b>	0	<b>6.5%</b>	<b>15.2%</b>	<b>1 998</b>
	40	2	0	37.9%	69.5%	3 600	<b>5</b>	0	<b>8.2%</b>	<b>15.2%</b>	<b>2 670</b>

622 and without adjustment mechanisms for each process, resulting in four struc-  
623 turally different design alternatives for each new product. This is similar to  
624 *Processes 2* and *3* in the design structure  $d_{SEP}$  in Figure 4.

625 The case study investigated the cross-effects of design alternative selec-  
626 tion and the level of automation in the assembly system. Specifically, three  
627 scenarios were studied, with only machine (M), only human (H) and mixed  
628 human and machine resources (H/M), respectively. The human adjustment  
629 precision of 0.15 mm is sufficiently high to assemble any of the design alterna-  
630 tives manually, and the processes are not automated at the beginning of the  
631 planning horizon in any of the scenarios. This can easily lead to higher au-  
632 tomation costs in M and H/M scenarios. The main findings of the case study  
633 are summarized in Figure 6 and Table 5, where the optimal solutions for the  
634 three scenarios are compared regarding their costs. Also, Table 6 shows the  
635 final automation status and the required precision of all production lines and  
636 processes in the three scenarios.

637 The overall costs are the highest in the *only machine* case, due to the  
638 high investment costs related to automation and precision upgrade: its cost  
639 is 15.56% higher than in the *only human* case, which is the baseline scenario  
640 and the current industrial practice. Total production costs are the lowest  
641 in the most generic H/M case, with a cost reduction of 13.47% compared  
642 to the baseline. This is the result of maximizing the investment efficiency  
643 by harmonizing product designs and the level of automation, i.e., using low-  
644 cost automated assembly for less precise products while assigning qualified

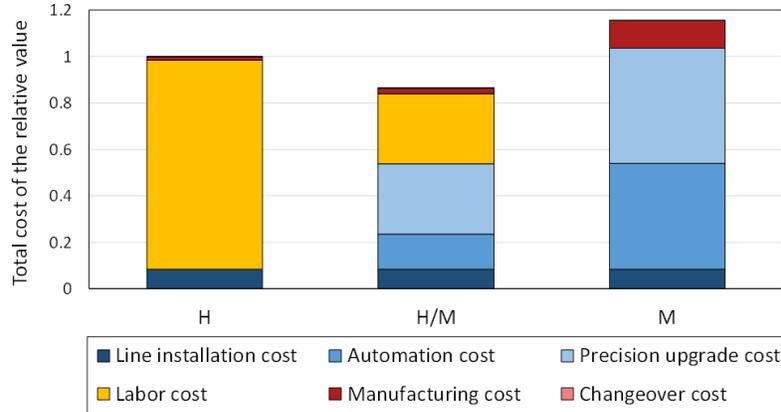


Figure 6: Results of the case study: comparison of optimal solutions under different resource selection options (H: only human, H/M: mixed human and machine resources, M: only machine).

645 workforce to high-precision products.

646 The design alternatives selected for the two new products are shown in  
 647 Table 7. In the conventional H case with highly qualified assembly workforce,  
 648 design alternatives with adjustment mechanisms are selected; this way, loose  
 649 tolerances are required on the parts, and hence, manufacturing costs can be  
 650 kept low. On the other hand, in the M case, the precision upgrade cost of  
 651 the machines is very high, and therefore, a proposal is adopted to reduce the  
 652 investments by selecting design alternatives without adjustments, yet, at the  
 653 price of higher manufacturing costs for more precise parts. In the H/M case,

Table 5: Results of the case study: cost structure in percentage of the total cost of the baseline H scenario.

	H	H/M	M
Line installation cost	8.27%	8.27%	8.27%
Automation cost	0.00%	15.27%	45.81%
Precision upgrade cost	0.00%	30.23%	49.60%
Labor cost	90.21%	30.30%	0.00%
Manufacturing cost	1.48%	2.39%	11.84%
Changeover cost	0.03%	0.07%	0.03%
Total	100.00%	86.53%	115.56%

Table 6: Final automation status and adjustment precision of each process for the different scenarios (H: operated by human worker with a fixed precision of 0.15 mm; M: operated by an automated machine with precision displayed in parentheses).

		H	H/M	M
Line 1	Process 1	H(0.15)	H(0.15)	M(0.15)
	Process 2	H(0.15)	H(0.15)	M(0.20)
Line 2	Process 1	H(0.15)	M(0.30)	M(0.30)
	Process 2	H(0.15)	M(0.30)	M(0.30)
Line 3	Process 1	H(0.15)	M(0.15)	M(0.15)
	Process 2	H(0.15)	M(0.20)	M(0.20)
Line 4	Process 1	H(0.15)	H(0.15)	M(0.56)
	Process 2	H(0.15)	H(0.15)	M(0.56)
Line 5	Process 1	H(0.15)	H(0.15)	M(0.56)
	Process 2	H(0.15)	M(0.56)	M(0.56)

Table 7: Design alternatives selected for the two new products in each scenario, for each process (w: with adjustment mechanism, w/o: without adjustment mechanism).

		H	H/M	M
New product 1	Process 1	w	w	w/o
	Process 2	w	w	w/o
New product 2	Process 1	w	w	w/o
	Process 2	w	w/o	w/o

654 the best compromise between the above extremities is derived with partial  
655 automation, and accordingly, with adjustment in a part of the dimension  
656 chains. This shows that joint optimization of the tolerance design and the  
657 assembly resource configuration enables finding the best compromise between  
658 the costs of manufacturing precise parts, applying qualified workforce for  
659 assembly, and investing into new assembly equipment to minimize the total  
660 production cost.

## 661 6. Discussion on practical application potential

662 The proposed methods were implemented in a decision support system  
663 consisting of three key modules. The *optimizer* implements the mathematical  
664 model and the two proposed solution approaches. This module supports the  
665 quantitative investigation of different scenarios. Numerical results can be  
666 passed to other modules for further processing. The purpose of the *Web UI*  
667 visualization module is to display the results of the optimizer and support

668 their analysis from all relevant aspects with the help of an interactive user  
669 interface presenting various types of charts, in an easy-to-understand format.

670 Finally, the design module was developed to facilitate the design workflow  
671 with the automation of design alternative generation and input data prepara-  
672 tion, as well as result visualization, all linked to a CAD environment. In  
673 this module, a master CAD model was prepared for this particular product  
674 family. The master model contains each existing structural design alternative  
675 type (i.e., CAD assembly models that differ in geometry beyond dimension or  
676 tolerance values), with the corresponding adjustable dimensional parameters  
677 (nominal and tolerance values). When establishing a new design alterna-  
678 tive, the designer can simply select the proper model configuration, fill in  
679 the assigned parameter values and create a new product (design alternative)  
680 instance.

681 As the CAD model of the new design alternative, including the toler-  
682 ance model, is built up automatically, the dimension chain and tolerance  
683 parameters of the new product variant can be exported and forwarded to  
684 the optimizer. This eliminates the manual preparation of tolerance design  
685 related data, and thus reduces the possibility of faulty input for the opti-  
686 mizer. Furthermore, the design module is capable of reading the solution  
687 computed by the optimizer, and therefore, the resulting tolerance allocation  
688 can be displayed on the original CAD model by actual geometry modifica-  
689 tion. This provides the designer with an effective tool to ensure the feasibility  
690 and correctness of the design. The connection between the three modules is  
691 depicted in Figure 7.

## 692 **7. Conclusions and future work**

693 Despite the various approaches proposed in literature and realized in the  
694 industrial practice for each phase of the product development process, find-  
695 ing the best tradeoff between product design and process/production plan  
696 efficiency is becoming more challenging than ever in multi-product assembly.  
697 This paper proposed a novel method for integrating and optimizing product  
698 design and process/production planning in order to maximize the investment  
699 efficiency and reduce the overall production cost. As an early step towards  
700 this goal, the paper focused on tolerances crucial for high-precision assem-  
701 bled products. Accordingly, a novel optimization problem was formulated  
702 that combines tolerance design, as the relevant sub-problem of product de-  
703 sign, with assembly resource configuration, as the corresponding sub-problem

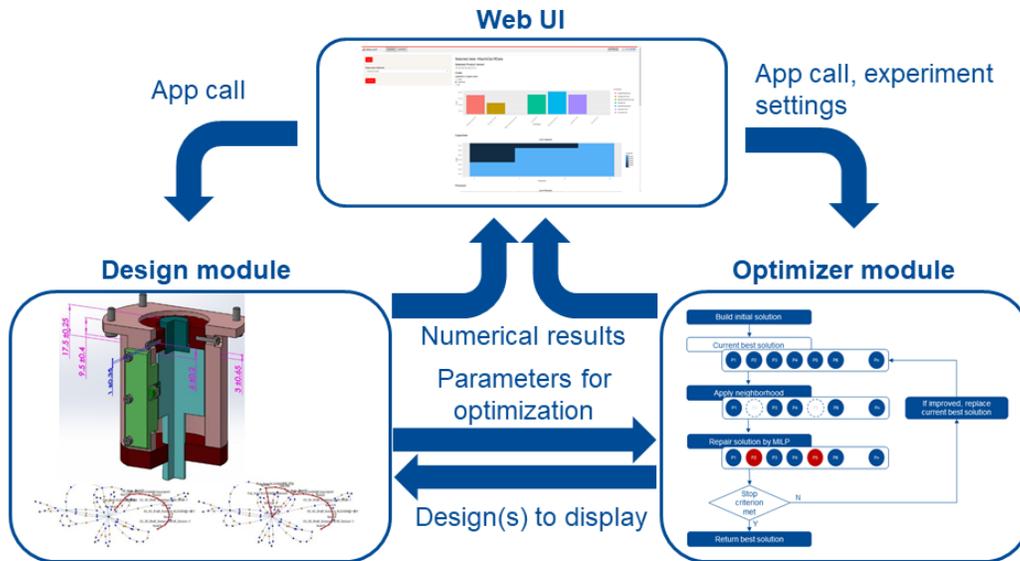


Figure 7: System architecture of the proposed decision support system.

704 in process and production planning.

705 The overall problem was formulated as a MILP, and an LNS matheuristic  
 706 solution method was proposed for solving large, industrially relevant  
 707 instances. The computational efficiency of LNS was demonstrated on a set  
 708 of instances based on industrial data. Furthermore, the relevance of the  
 709 proposed problem to industry was shown in a case study that focused on  
 710 evaluating different design alternatives and different combinations of human  
 711 and machine resources. It was confirmed that an appropriate combination of  
 712 different resources and a corresponding selection of design alternatives can  
 713 minimize the total production costs.

714 As a direction for future research, it is necessary to evaluate the robustness  
 715 of the approach to the fluctuation of the long-term production plan and  
 716 the various cost functions that define the input of the model. A relevant  
 717 extension of the model will cover the case of selective assembly instead of full  
 718 interchangeability of the parts. The implementation of the proposed decision  
 719 support system with the optimization engine, a web-based dashboard, and  
 720 3D CAD integration is in progress, in order to promote future utilization in  
 721 the industry.

722 **Acknowledgments**

723 The Hungarian authors of the paper were supported by the ED\_18-2-2018-  
724 0006 grant on “Research on prime exploitation of the potential provided  
725 by the industrial digitalisation” and GINOP-2.3.2-15-2016-00002 grant on  
726 an “Industry 4.0 research and innovation center of excellence”. A. Kovács  
727 acknowledges the support of the János Bolyai Research Fellowship.

- 728 [1] S. J. Hu, J. Ko, L. Weyand, H. ElMaraghy, T. Lien, Y. Koren, H. Bley,  
729 G. Chryssolouris, N. Nasr, M. Shpitalni, Assembly system design and  
730 operations for product variety, *CIRP Annals–Manufacturing Technology*  
731 60 (2) (2011) 715–733.
- 732 [2] J. Daaboul, C. Da Cunha, A. Bernard, F. Laroche, Design for mass  
733 customization: Product variety vs. process variety, *CIRP Annals–*  
734 *Manufacturing Technology* 60 (1) (2011) 169–174.
- 735 [3] H. Wang, X. Zhu, H. Wang, S. J. Hu, Z. Lin, G. Chen, Multi-objective  
736 optimization of product variety and manufacturing complexity in mixed-  
737 model assembly systems, *Journal of Manufacturing Systems* 30 (1)  
738 (2011) 16–27.
- 739 [4] A.-L. Andersen, T. D. Brunoe, K. Nielsen, C. Rösiö, Towards a generic  
740 design method for reconfigurable manufacturing systems: Analysis and  
741 synthesis of current design methods and evaluation of supportive tools,  
742 *Journal of Manufacturing Systems* 42 (2017) 179–195.
- 743 [5] E. Lutters, F. J. van Houten, A. Bernard, E. Mermoz, C. S. Schutte,  
744 Tools and techniques for product design, *CIRP Annals–Manufacturing*  
745 *Technology* 63 (2) (2014) 607–630.
- 746 [6] T. Tolio, D. Ceglarek, H. ElMaraghy, A. Fischer, S. Hu, L. Laperrière,  
747 S. T. Newman, J. Váncza, SPECIES—Co-evolution of products, pro-  
748 cesses and production systems, *CIRP Annals–Manufacturing Technol-*  
749 *ogy* 59 (2) (2010) 672–693.
- 750 [7] H. ElMaraghy, G. Schuh, W. ElMaraghy, F. Piller, P. Schönsleben,  
751 M. Tseng, A. Bernard, Product variety management, *CIRP Annals–*  
752 *Manufacturing Technology* 62 (2) (2013) 629–652.

- 753 [8] G. Boothroyd, P. Dewhurst, Product design for manufacture and assem-  
754 bly, Boothroyd Dewhurst, Inc., 1989.
- 755 [9] J. He, Tolerancing for manufacturing via cost minimization, Interna-  
756 tional Journal of Machine Tools and Manufacture 31 (4) (1991) 455–470.
- 757 [10] D. Tsutsumi, D. Gyulai, A. Kovács, B. Tipary, Y. Ueno, Y. Nonaka,  
758 L. Monostori, Towards joint optimization of product design, process  
759 planning and production planning in multi-product assembly, CIRP  
760 Annals–Manufacturing Technology 67 (1) (2018) 441–446.
- 761 [11] F. Hansen, Konstruktionssystematic Grundlagen für eine allgemeine  
762 Konstruktionslehre, VEB Verlag Technik, Berlin, 1965.
- 763 [12] J. Jänsch, H. Birkhofer, The development of the guideline VDI 2221–The  
764 change of direction, in: Proceedings DESIGN 2006, the 9th International  
765 Design Conference, 2006, pp. 45–52.
- 766 [13] G. Q. Huang, Design for X: Concurrent Engineering Imperatives,  
767 Springer, 1996.
- 768 [14] S. Miyakawa, T. Ohashi, The Hitachi assemblability evaluation method  
769 (AEM), Proceedings of the International Conference on Product Design  
770 for Assembly (1986).
- 771 [15] B. Miles, K. Swift, Working together, Boothroyd Dewhurst, Inc., 1992.
- 772 [16] A. Redford, J. Chal, Design for Assembly, Principle and Practice, Mc-  
773 GrawHill Book Company Europe, 1994.
- 774 [17] G. Molcho, Y. Zipori, R. Schneor, O. Rosen, D. Goldstein, M. Shpi-  
775 talni, Computer aided manufacturability analysis: Closing the knowl-  
776 edge gap between the designer and the manufacturer, CIRP Annals–  
777 Manufacturing Technology 57 (1) (2008) 153–158.
- 778 [18] A. Tornow, R. Graubohm, F. Dietrich, K. Dröder, Design automation  
779 for battery system variants of electric vehicles with integrated product  
780 and process evaluation, Procedia CIRP 50 (2016) 424–429.
- 781 [19] D. Whitney, Mechanical Assemblies: Their Design, Manufacture, and  
782 Role in Product Development, Oxford University Press, 2004.

- 783 [20] B. Heling, A. Aschenbrenner, M. Walter, S. Wartzack, On connected  
784 tolerances in statistical tolerance-cost-optimization of assemblies with  
785 interrelated dimension chains, *Procedia CIRP* 43 (2016) 262–267.
- 786 [21] P. K. Singh, S. C. Jain, P. K. Jain, Advanced optimal tolerance design  
787 of mechanical assemblies with interrelated dimension chains and process  
788 precision limits, *Computers in Industry* 56 (2) (2005) 179–194.
- 789 [22] F. Schlather, V. Hoesl, F. Oefele, M. F. Zaeh, Tolerance analysis of  
790 compliant, feature-based sheet metal structures for fixtureless assembly,  
791 *Journal of Manufacturing Systems* 49 (2018) 25–35.
- 792 [23] L. Andolfatto, F. Thiébaud, C. Lartigue, M. Douilly, Quality- and cost-  
793 driven assembly technique selection and geometrical tolerance allocation  
794 for mechanical structure assembly, *Journal of Manufacturing Systems*  
795 33 (1) (2014) 103–115.
- 796 [24] J. G. Li, Y. X. Yao, P. Wang, Assembly accuracy prediction based  
797 on CAD model, *The International Journal of Advanced Manufacturing*  
798 *Technology* 75 (5) (2014) 825–832.
- 799 [25] J.-Y. Chiang, T.-R. Tsai, Y. Lio, W. Lu, D. Shi, An integrated approach  
800 for the optimization of tolerance design and quality cost, *Computers &*  
801 *Industrial Engineering* 87 (2015) 186–192.
- 802 [26] S. Xu, J. Keyser, Statistical geometric computation on tolerances for  
803 dimensioning, *Computer-Aided Design* 70 (2016) 193–201.
- 804 [27] Y. Zhong, Y. Qin, M. Huang, W. Lu, W. Gao, Y. Du, Automatically  
805 generating assembly tolerance types with an ontology-based approach,  
806 *Computer-Aided Design* 45 (11) (2013) 1253–1275.
- 807 [28] S. Ghandi, E. Masehian, Review and taxonomies of assembly and disas-  
808 sembly path planning problems and approaches, *Computer-Aided De-*  
809 *sign* 67–68 (2015) 58–86.
- 810 [29] X. Chen, S. Gao, Y. Yang, S. Zhang, Multi-level assembly model for top-  
811 down design of mechanical products, *Computer-Aided Design* 44 (10)  
812 (2012) 1033–1048.

- 813 [30] S. Rachuri, Y.-H. Han, S. Foufou, S. C. Feng, U. Roy, F. Wang, R. D.  
814 Sriram, K. W. Lyons, A model for capturing product assembly infor-  
815 mation, *Journal of Computing and Information Science in Engineering*  
816 6 (1) (2006) 11–21.
- 817 [31] Z. Ding, B. Hon, Constraints analysis and evaluation of manual assem-  
818 bly, *CIRP Annals–Manufacturing Technology* 62 (1) (2013) 1–4.
- 819 [32] G. Fantoni, M. Santochi, G. Dini, K. Tracht, B. Scholz-Reiter, J. Fleis-  
820 cher, T. K. Lien, G. Seliger, G. Reinhart, J. Franke, H. N. Hansen,  
821 A. Verl, Grasping devices and methods in automated production pro-  
822 cesses, *CIRP Annals–Manufacturing Technology* 63 (2) (2014) 679–701.
- 823 [33] B. W. Pearce, K. Antani, L. Mears, K. Funk, M. E. Mayorga, M. E.  
824 Kurz, An effective integer program for a general assembly line balancing  
825 problem with parallel workers and additional assignment restrictions,  
826 *Journal of Manufacturing Systems* 50 (2019) 180–192.
- 827 [34] A. Gomez, J. Rios, F. Mas, A. Vizan, Method and software application  
828 to assist in the conceptual design of aircraft final assembly lines, *Journal*  
829 *of Manufacturing Systems* 40 (2016) 37–53.
- 830 [35] X. Fiorentini, I. Gambino, V.-C. Liang, S. Rachuri, M. Mani, C. Bock,  
831 An ontology for assembly representation, *National Institute of Standards*  
832 *and Technology* (2007).
- 833 [36] Y. Pochet, L. A. Wolsey, *Production Planning by Mixed Integer Pro-*  
834 *gramming*, Springer, 2010.
- 835 [37] V. Maniezzo, T. Stützle, S. Voß, *Matheuristics–Hybridizing Metaheuris-*  
836 *tics and Mathematical Programming*, Springer, 2010.
- 837 [38] R. K. Ahuja, O. Ergun, J. B. Orlin, A. P. Punnen, A survey of very large-  
838 scale neighborhood search techniques, *Discrete Applied Mathematics*  
839 123 (1-3) (2002) 75102.
- 840 [39] S. Mouthuy, P. V. Hentenryck, Y. Deville, Constraint-based very large-  
841 scale neighborhood search, *Constraints* 17 (1) (2012) 87–122.
- 842 [40] P. Laborie, D. Godard, Self-adapting large neighborhood search: Ap-  
843 plication to single-mode scheduling problems, in: *Proceedings of*

- 844 MISTA'07, Multidisciplinary International Scheduling Conference: The-  
845 ory & Applications, 2007, pp. 276–284.
- 846 [41] P. M. Thompson, Local search algorithms for vehicle routing and other  
847 combinatorial problems, Ph.D. thesis, Massachusetts Institute of Tech-  
848 nology (1988).
- 849 [42] D. G. Ullman, The mechanical design process (4th Edition), McGraw-  
850 Hill Education New York, 2009.
- 851 [43] P. Belotti, P. Bonami, M. Fischetti, A. Lodi, M. Monaci, A. Nogales-  
852 Gómez, D. Salvagnin, On handling indicator constraints in mixed inte-  
853 ger programming, Computational Optimization and Applications 65 (3)  
854 (2016) 545–566.